Spin Hall Effect and Optical Detection in Semiconductor Quantum Wells

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Outline

- Introduction
- Nonlinear Rashba model and Spin Relaxation
- Spin Hall Effect in narrow band-gap QW
- Directly optical detection of pure spin current
- Conlusions

Collaborators：

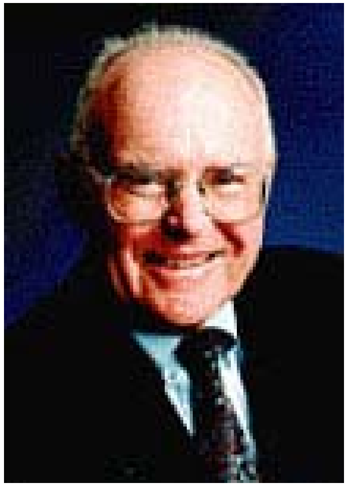
W. Yang, J. T. Liu, Z. Zhang, J. Li (SKLSM, IOS)

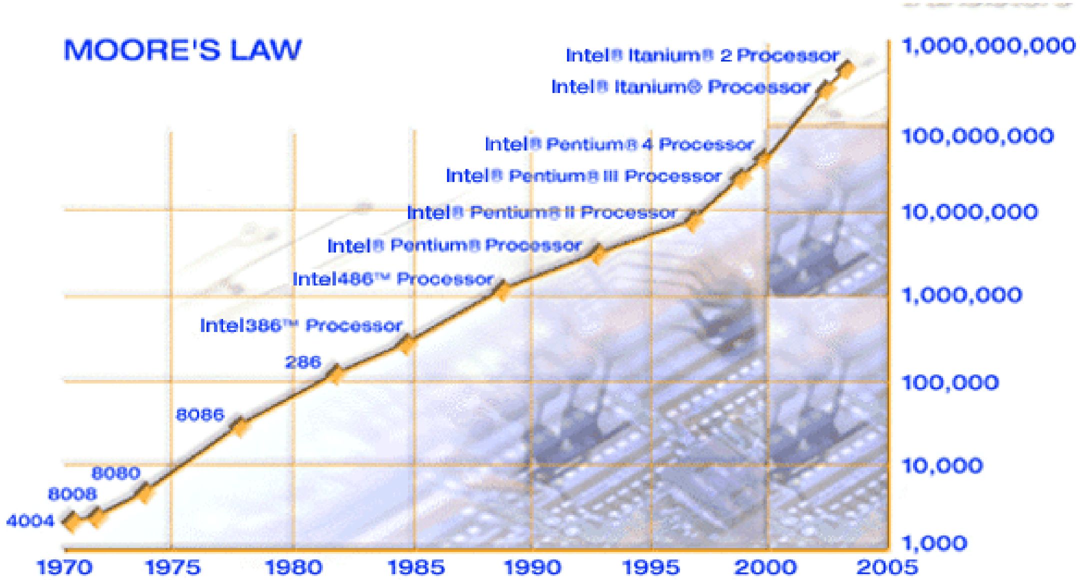
Prof. S. C. Zhang (Stanford University)

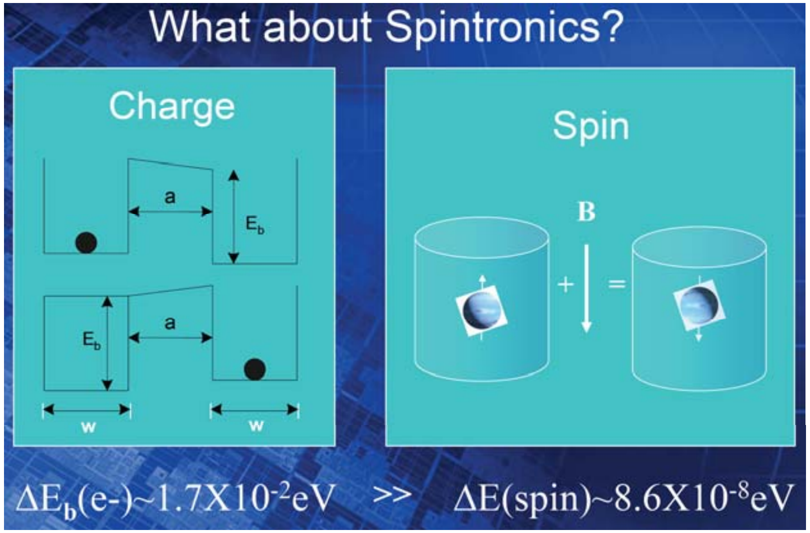
Acknowledgments: NSFC Grant No. 60525405 and

The innovation project of knowledge from CAS

Introduction...







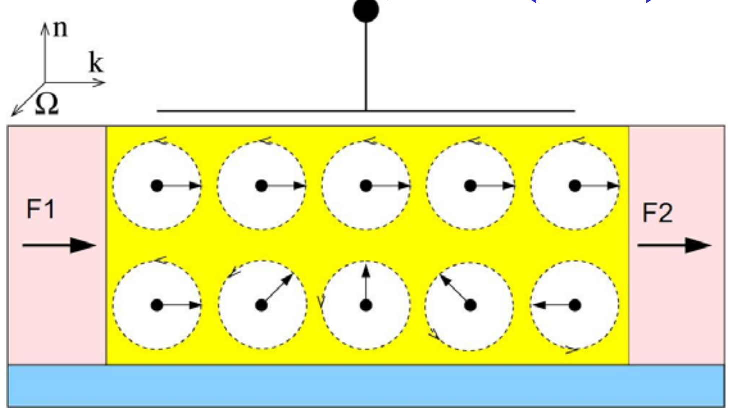
1、Power consumption---heat
2、Quantum effect

Possible solution: Spintronics
Limits: Room temperature spin injection and relaxation

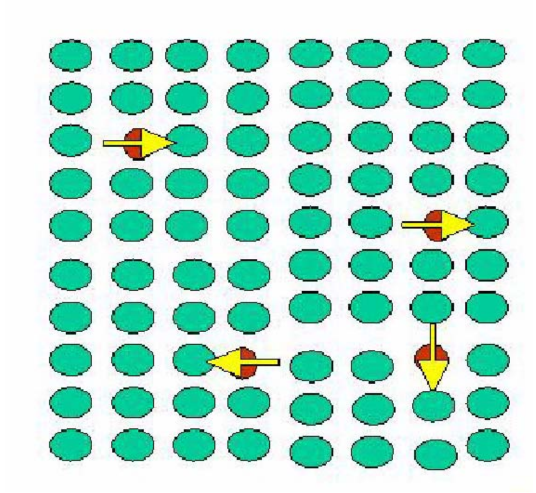
Manipulation of spin in Semiconductors

1, sp-d exchange interaction:
DMS, e.g., GaMnAs
2, Laser pulse:
Optical Stark effect
3, Relativistic effect:
Spin-orbit interaction

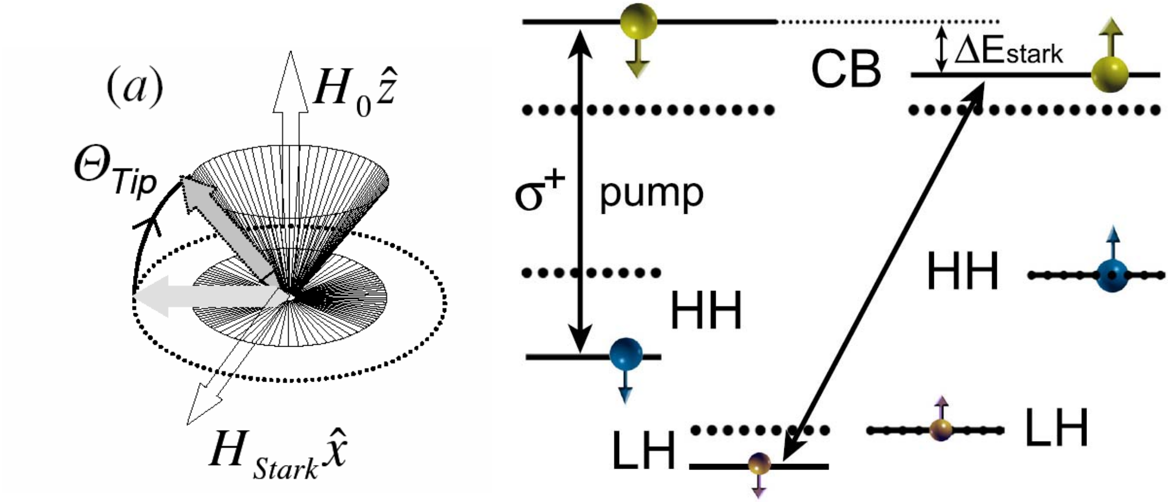
Datta-Das transistor, APL(1990)



DMS

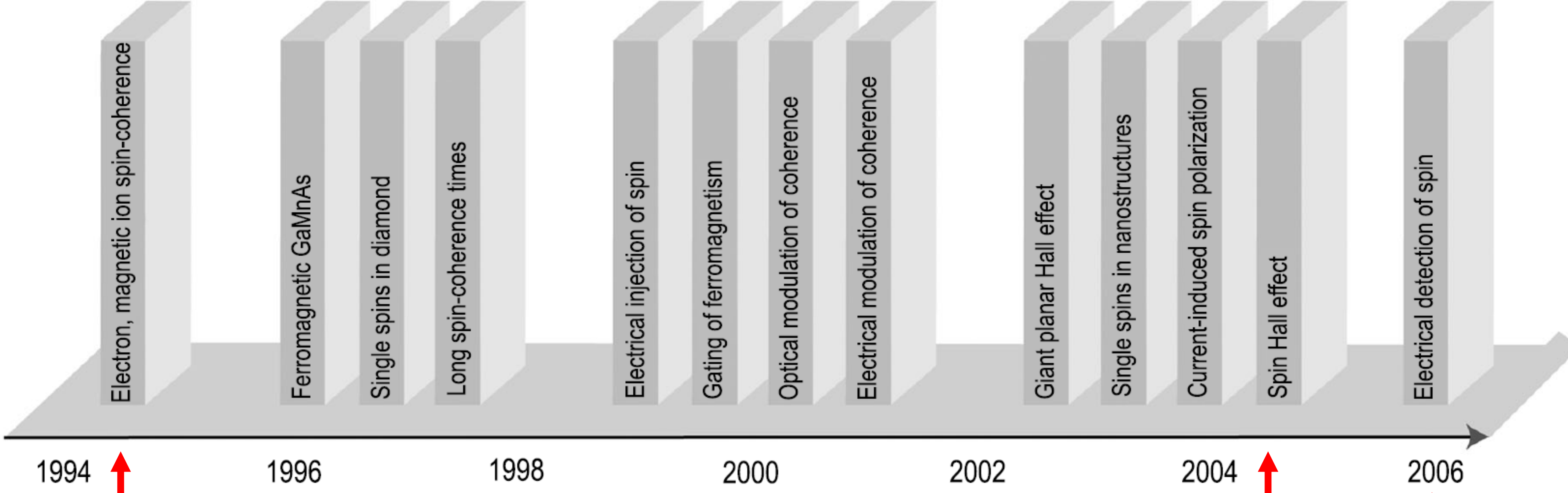


Optical Stark effect



Progress of spintronics

Milestones



From D. D. Awaschlom

Spin Hall Effect

Spin coherence

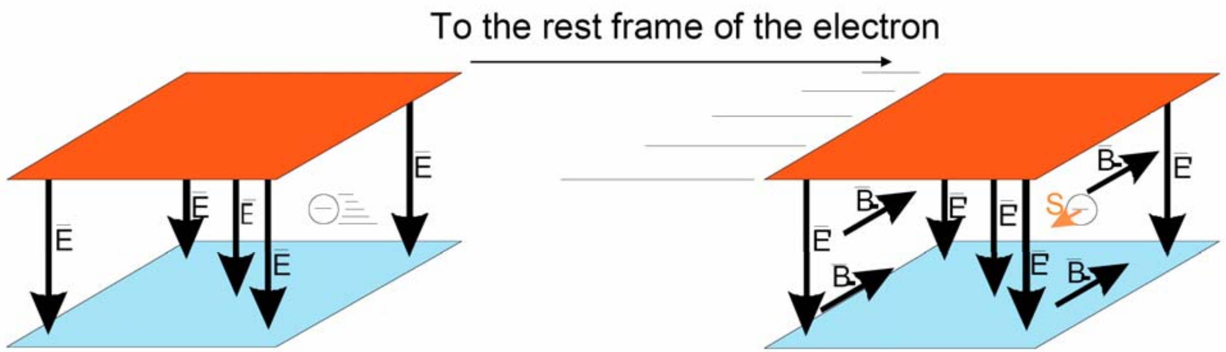
Detection of spin

The spin-orbit interaction

Origin of Rashba spin-orbit coupling

Relativistic effect

$$\vec{B}_{ef} = -\frac{1}{c^2}\vec{v}\times\vec{E}$$



$$H_{\mathrm{Dirac}} = \begin{bmatrix} mc^2 + V(\mathbf{r}) & c\boldsymbol{\sigma}\bullet\boldsymbol{\pi} \\ c\boldsymbol{\sigma}\bullet\boldsymbol{\pi} & -mc^2 + V(\mathbf{r}) \end{bmatrix} \rightarrow H_{\mathrm{eff}} = e^{S} H_{\mathrm{Dirac}} e^{-S} = \begin{bmatrix} H & 0 \\ 0 & \bar{H} \end{bmatrix}$$

orbital **spin**

$$H = mc^2 + V(\mathbf{r}) + \frac{\pi^2}{2m} + \mu_B \boldsymbol{\sigma}\bullet\mathbf{B} + \frac{\hbar^2}{4m^2c^2}(\nabla V \times \boldsymbol{\pi})\bullet\boldsymbol{\sigma} + \frac{\hbar^2}{8m^2c^2}\nabla^2 V - \frac{1}{8m^3c^2}(\boldsymbol{\sigma}\bullet\boldsymbol{\pi})^4$$

Symmetry

$$|\mathbf{x}\rangle \to |\mathbf{x}\rangle^{\pi} = |-\mathbf{x}\rangle,$$
$$|\mathbf{p}\rangle \to |\mathbf{p}\rangle^{\pi} = |-\mathbf{p}\rangle,$$
$$|j,m\rangle \to |j,m\rangle^{\pi} = |j,m\rangle$$

$$|\mathbf{x}\rangle \to |\mathbf{x}\rangle^{\theta} = |\mathbf{x}\rangle,$$
$$|\mathbf{p}\rangle \to |\mathbf{p}\rangle^{\theta} = |-\mathbf{p}\rangle,$$
$$|j,m\rangle \to |j,m\rangle^{\theta} = D(\mathbf{e}_y,-\pi)|j,m\rangle \propto |j,-m\rangle,$$

$$H|n\mathbf{k}s\rangle = E_{n\mathbf{k}s}|n\mathbf{k}s\rangle$$

Space inversion

$$H|n\mathbf{k}s\rangle^{\pi} = E_{n\mathbf{k}s}|n\mathbf{k}s\rangle^{\pi}$$
$$|n\mathbf{k}s\rangle^{\pi} = |n,-\mathbf{k},s\rangle$$
$$\Downarrow$$
$$E_{n\mathbf{k}s} = E_{n,-\mathbf{k},s}$$

Time reversal

$$H|n\mathbf{k}s\rangle^{\theta} = E_{n\mathbf{k}s}|n\mathbf{k}s\rangle^{\theta}$$
$$|n\mathbf{k}s\rangle^{\theta} \propto |n,-\mathbf{k},-s\rangle$$
$$\Downarrow$$
$$E_{n\mathbf{k}s} = E_{n,-\mathbf{k},-s}$$

With both symmetry

$$E_{n\mathbf{k}s} = E_{n,\mathbf{k},-s}$$

Spin splitting induced by the symmetry breaking

Spin-orbit interactions

E. I. Rashba, JETP 1961

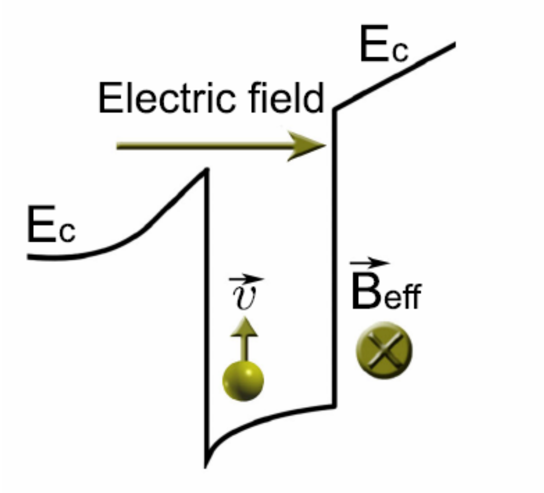
G. Dresselahus, Phys Rev 1955

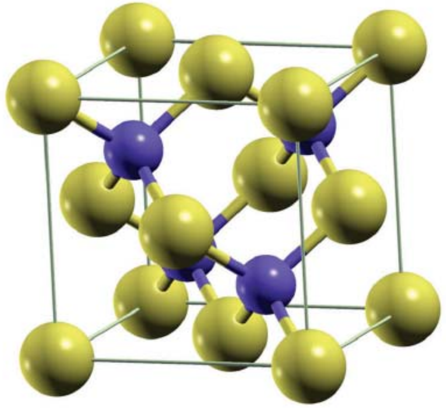
Rashba SOI

Structural asymmetry (SIA)

Dresselhaus SOI

Crystal inversion asymmetry(BIA)





$$H_R = \alpha\left[\bar{\sigma}\times\bar{k}\right]\cdot\hat{z} = \alpha\left(\sigma_x k_y - \sigma_y k_x\right)$$

$$\bar{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$$

$$H_D = \gamma\,\bar{\sigma}\cdot\bar{\kappa}$$

$$\kappa_x = k_y k_x k_y - k_z k_x k_z$$

$$\kappa_y = k_z k_y k_z - k_x k_y k_x$$

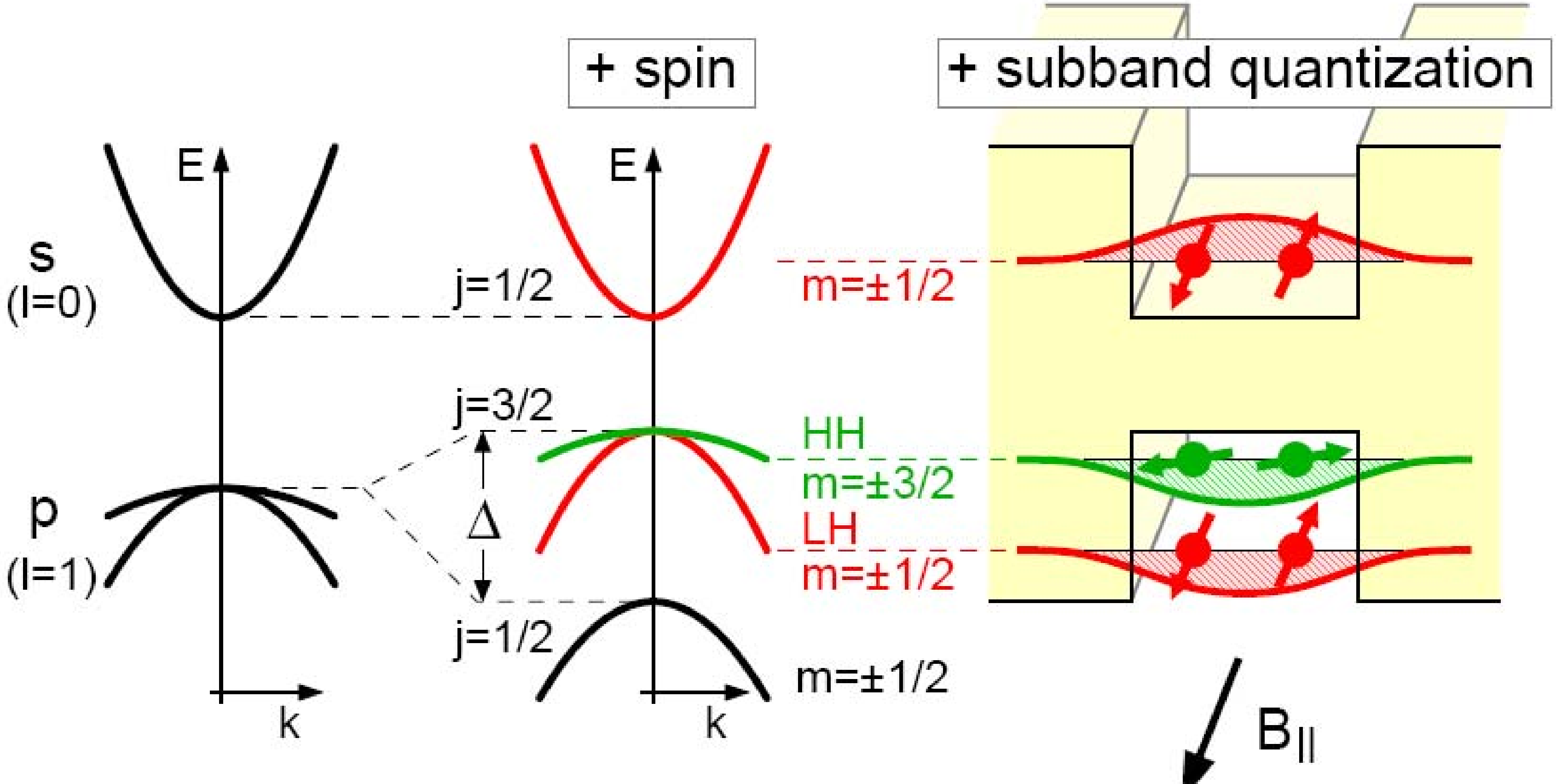
$$\kappa_z = k_x k_z k_x - k_y k_z k_y$$

R. Winkler, ***Spin-Orbit Coupling Effects in Two-Dimensional Electron and Hole System, Springer ,2003***

Linear Rashba model

Semiconductor quantum well

$$\mathcal{H}_{SO} \sim \sigma \cdot \mathbf{B}(\mathbf{k})$$

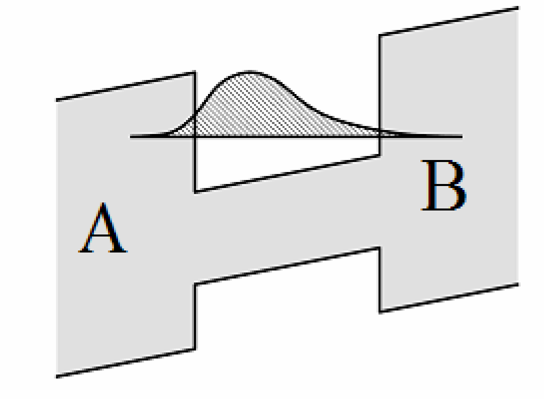


$\mathbf{B_R} = \alpha\,(\mathbf{k} \times \mathbf{z})$

Linear Rashba model

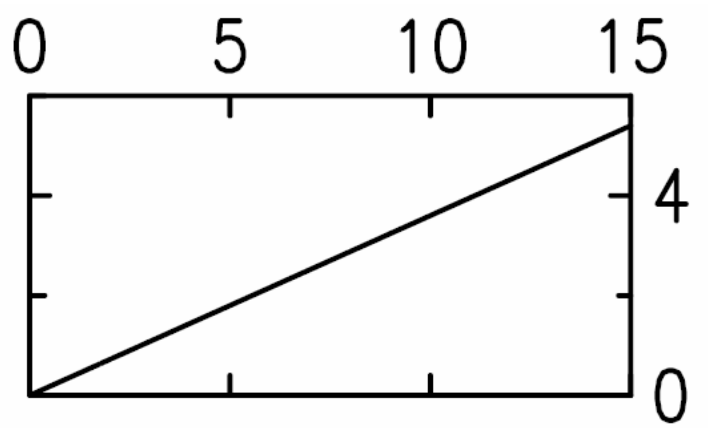
$$H_R = \alpha\left[\bar{\sigma}\times\bar{k}\right]\cdot\hat{z} = \alpha\left(\sigma_x k_y - \sigma_y k_x\right)$$

$$\bar{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$$



Spin splitting

$$\Delta E = 2\alpha k_{//}$$



R. Winkler and U. Rössler, PRB 48, 8918 (1993)

E. A. de Andrada e Silva et al., PRB 55, 16293 (1997)

R. Winkler and U. Rössler, PRB 62, 4245 (2000)

S. Lamari, PRB 64, 245340 (2001)

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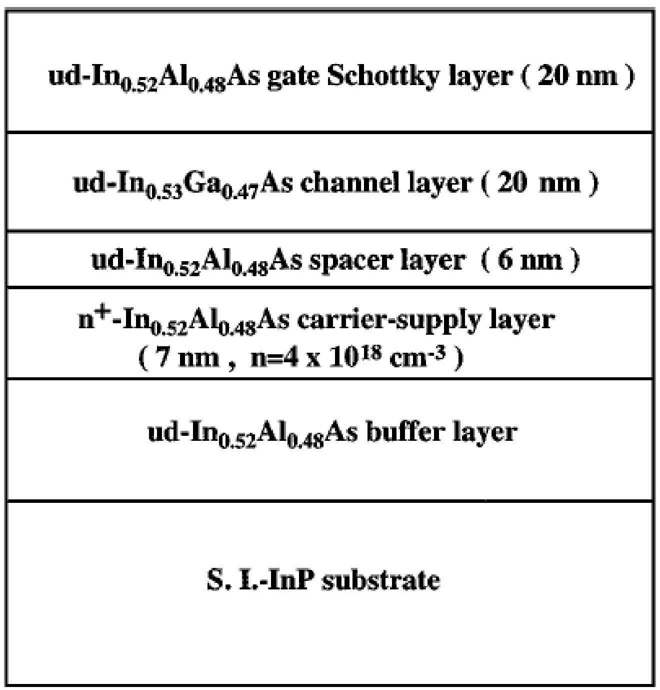
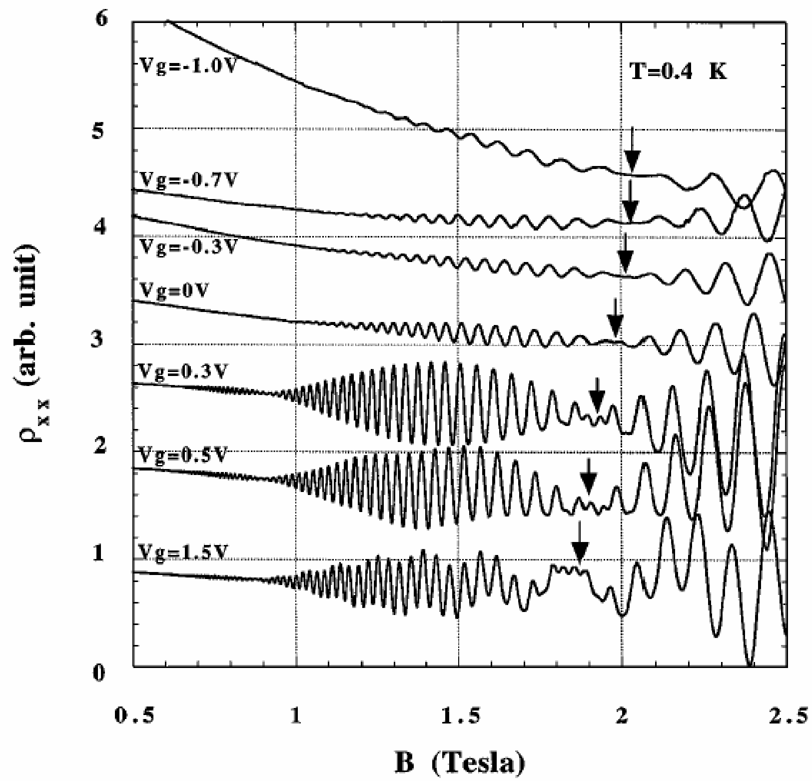
Experiment

Gate Control of Spin-Orbit Interaction in an Inverted $In_{0.53}Ga_{0.47}As/In_{0.52}Al_{0.48}As$ Heterostructure

Junsaku Nitta, Tatsushi Akazaki, and Hideaki Takayanagi
NTT Basic Research Laboratories, 3-1 Wakamiya, Morinosato, Atsugi-shi, Kanagawa 243-01, Japan

Takatomo Enoki
NTT System Electronics Laboratories, 3-1 Wakamiya, Morinosato, Atsugi-shi, Kanagawa 243-01, Japan
(Received 23 July 1996)





Effective-mass theory

Effective mass k·p theory

the Bloch functions $e^{i\boldsymbol{k}\cdot\boldsymbol{r}} u_{\nu\boldsymbol{k}}(\boldsymbol{r}) \equiv e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \langle \boldsymbol{r} | \nu \boldsymbol{k} \rangle$

$$\left[\frac{p^2}{2m_0} + V_0(\boldsymbol{r})\right] e^{i\boldsymbol{k}\cdot\boldsymbol{r}} u_{\nu\boldsymbol{k}}(\boldsymbol{r}) = E_\nu(\boldsymbol{k})\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} u_{\nu\boldsymbol{k}}(\boldsymbol{r}) \,.$$

$$\left[\frac{p^2}{2m_0} + V_0 + \frac{\hbar^2 k^2}{2m_0} + \frac{\hbar}{m_0}\boldsymbol{k}\cdot\boldsymbol{p}\right] |\nu\boldsymbol{k}\rangle = E_\nu(\boldsymbol{k})\, |\nu\boldsymbol{k}\rangle$$

Considering SOI

$$[\frac{p^2}{2m_0} + V_0 + \frac{k^2}{2m_0} + \frac{\hbar}{m_0} k \bullet \pi + \frac{\hbar}{4m^2{}_0 c^2} p \bullet (\sigma \times \nabla V)] \left| nk \right\rangle = E_n \left| nk \right\rangle,$$

$$where \quad \pi = p + \frac{\hbar}{4m^2{}_0 c^2} \sigma \times \nabla V_0,$$

$$\left| nk \right\rangle = \sum_{\nu,\sigma=\uparrow\downarrow} c_{\nu,\sigma'} \left| nk_0 \right\rangle \otimes \left| \sigma' \right\rangle$$

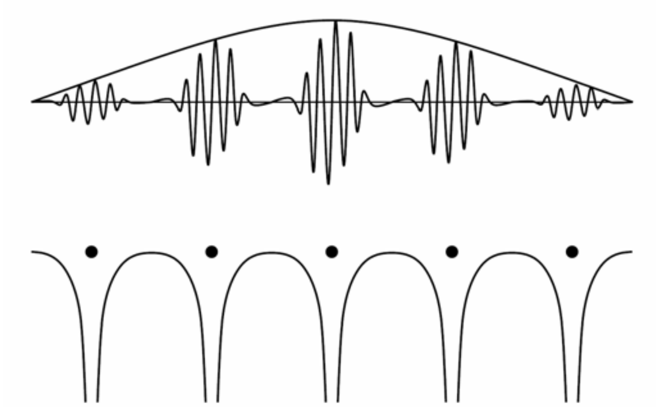
Effective-mass theory

Effective mass k·p theory

$$\sum_{\nu'\sigma'}\left\{\left[E_{\nu'}(0)+\frac{k^2}{2m_0}\right]\delta_{\nu\nu'}\delta_{\sigma\sigma'}+\frac{\hbar}{m_0}\vec{k}\bullet\vec{P}_{\nu\nu'\sigma\sigma'}+\Delta_{\nu\nu'\sigma\sigma'}\right\}c_{n\nu'\sigma'}(k)=E_n(k)c_{n\nu'\sigma'}(k),$$

where $P_{\nu\nu'\sigma\sigma'}=\langle\nu\sigma|\pi|\nu'\sigma'\rangle,$

$$\Delta_{\nu\nu'\sigma\sigma'}=\frac{\hbar}{4m_0^{\,2}c^2}\langle\nu\sigma|p\bullet(\sigma\times\nabla V)|\nu'\sigma'\rangle,$$

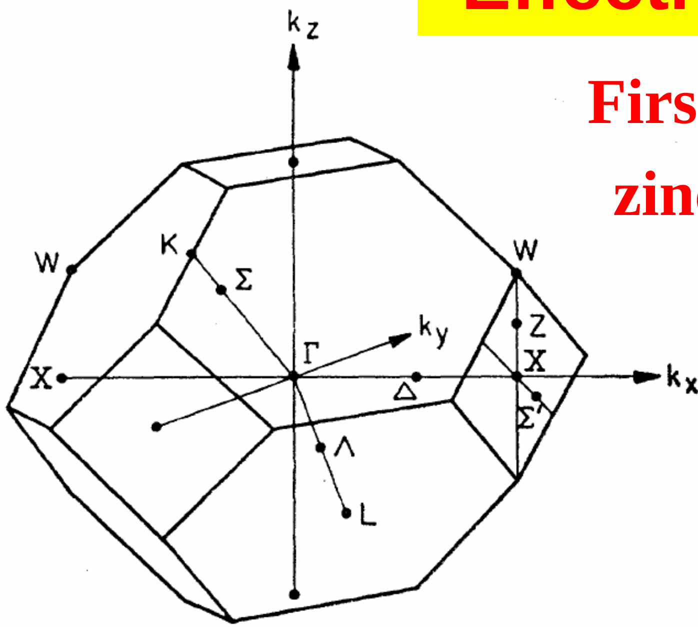


The envelope function approximation

$$\Psi(r)=\sum_{\nu'\sigma'}\psi_{\nu'\sigma'}(r)u_{\nu'0}(r)|\sigma'\rangle$$

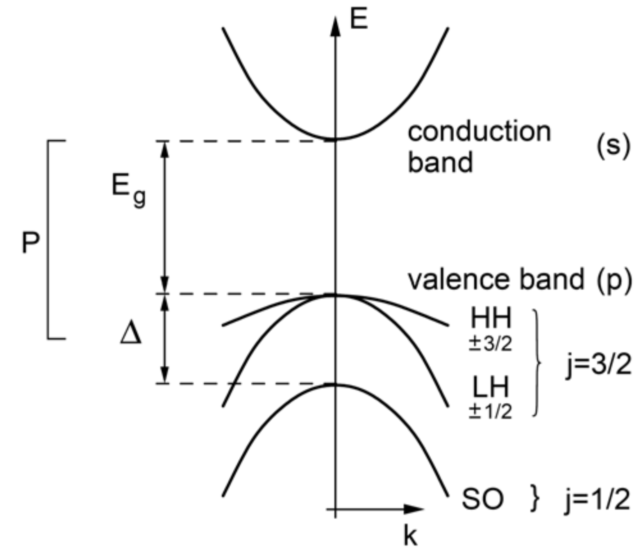
$$\sum_{\nu'\sigma'}\left\{\left[E_{\nu'}(0)+\frac{k^2}{2m_0}\right]\delta_{\nu\nu'}\delta_{\sigma\sigma'}+\frac{\hbar}{m_0}\vec{k}\bullet\vec{P}_{\nu\nu'\sigma\sigma'}+\Delta_{\nu\nu'\sigma\sigma'}\right\}\psi_{\nu'\sigma'}(r)=E_n(k)\psi_{\nu\sigma}(r),$$

Effective-mass theory



First Brillouin Zone of zinc-blende structure

T_d group



$\Psi_1 = | S \uparrow >$

$\Psi_2 = | S \downarrow >$

$\Psi_3 = | 3/2, 3/2 > = 1/\sqrt{2} \, | X + iY, \uparrow >$

$\Psi_4 = | 3/2, 1/2 > = i/\sqrt{6} [| X + iY, \downarrow > - 2 | Z, \uparrow >]$

$\Psi_5 = | 3/2, -1/2 > = i/\sqrt{6} [| X - iY, \uparrow > + 2 | Z, \downarrow >]$

$\Psi_6 = | 3/2, -3/2 > = 1/\sqrt{2} \, | X - iY, \downarrow >$

$\Psi_7 = | 1/2, 1/2 > = 1/\sqrt{3} [| X + iY, \downarrow > + | Z, \uparrow >]$

$\Psi_8 = | 1/2, -1/2 > = i/\sqrt{3} [- | X - iY, \uparrow > - | Z, \downarrow >]$

Basis

$\Psi_k = u_k(r) e^{ikr}$

$$H = p^2 / 2m_0 + V + \frac{\hbar}{4m^2c^2} (\nabla V \times p) \cdot \sigma$$

H_{SO}

Origin of Rashba spin-orbit coupling

$$H_{\mathsf{k}} = \begin{bmatrix} \mathsf{k}A_c\mathsf{k} & 0 & iP_0k_+/\sqrt{2} & \sqrt{2/3}P_0k_z & iP_0k_-/\sqrt{6} & 0 & iP_0k_z/\sqrt{3} & P_0k_-/\sqrt{3} \\ & \mathsf{k}A_c\mathsf{k} & 0 & -P_0k_+/\sqrt{6} & i\sqrt{2/3}P_0k_z & -P_0k_-/\sqrt{2} & iP_0k_+/\sqrt{3} & -P_0k_z/\sqrt{3} \\ & & P+Q & L-i\sqrt{6}S & M & 0 & iL/\sqrt{2}+\sqrt{3}S & -i\sqrt{2}M \\ & & & P-Q & -2\sqrt{2}iS & M & -i\sqrt{2}Q & i\sqrt{3/2}L-S \\ & & & & P-Q & -L-i\sqrt{6}S & -i\sqrt{3/2}L^{+}+S^{+} & -i\sqrt{2}Q \\ & & & & & P-Q & -i\sqrt{2}M^{+} & -iL^{+}/\sqrt{2}-\sqrt{3}S^{+} \\ & & & & & & P & 2\sqrt{2}iS \\ & & & & & & & P \end{bmatrix},$$

$$H_{\text{eff}}(\mathbf{k}_{//}) = E_c(z)+V(z)+\mathbf{k}\frac{\hbar^2}{2m^*(z)}\mathbf{k}+\alpha_0(z)(\mathbf{k}_{//}\times\mathbf{e}_z)\cdot\sigma,$$

$$\frac{m_0}{m^*(z)} = \gamma_c + \frac{2E_p}{3U_{\text{lh}}} + \frac{E_p}{3U_{\text{so}}},$$

$$\alpha_0(z) = \hbar^2/(6m_0)\partial\gamma(z)/\partial z$$

$$\gamma(z) = E_p\cdot(1/U_{lh} - 1/U_{so});$$

$$U_{lh} = E - H_{lh};$$

$$U_{so} = E - H_{so};$$

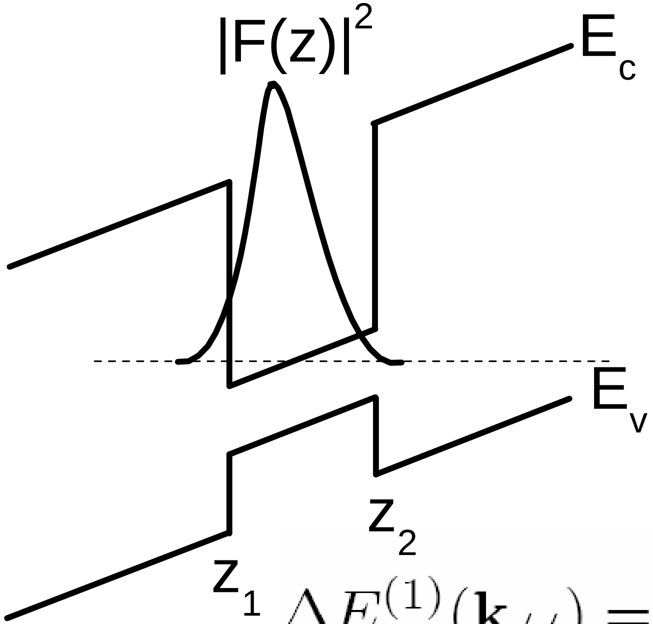
$$H_{lh} = E_v - V + P + Q;$$

$$H_{so} = E_v - \Delta + V + P.$$

PHYS. REV. B 73, 113303 (2006)

W. Yang and Kai Chang

Zero-field Rashba spin splitting



$$\Delta E_n(\mathbf{k}_{//}) = \Delta E_n^{(1)}(\mathbf{k}_{//}) + \Delta E_n^{(2)}(\mathbf{k}_{//}),$$

$$\Delta E_n^{(1)}(\mathbf{k}_{//}) = \frac{\hbar^2}{3m_0} k_{//} \sum_j |F_n(z_j)|^2 \left[\gamma(z_j^+) - \gamma(z_j^-)\right],$$

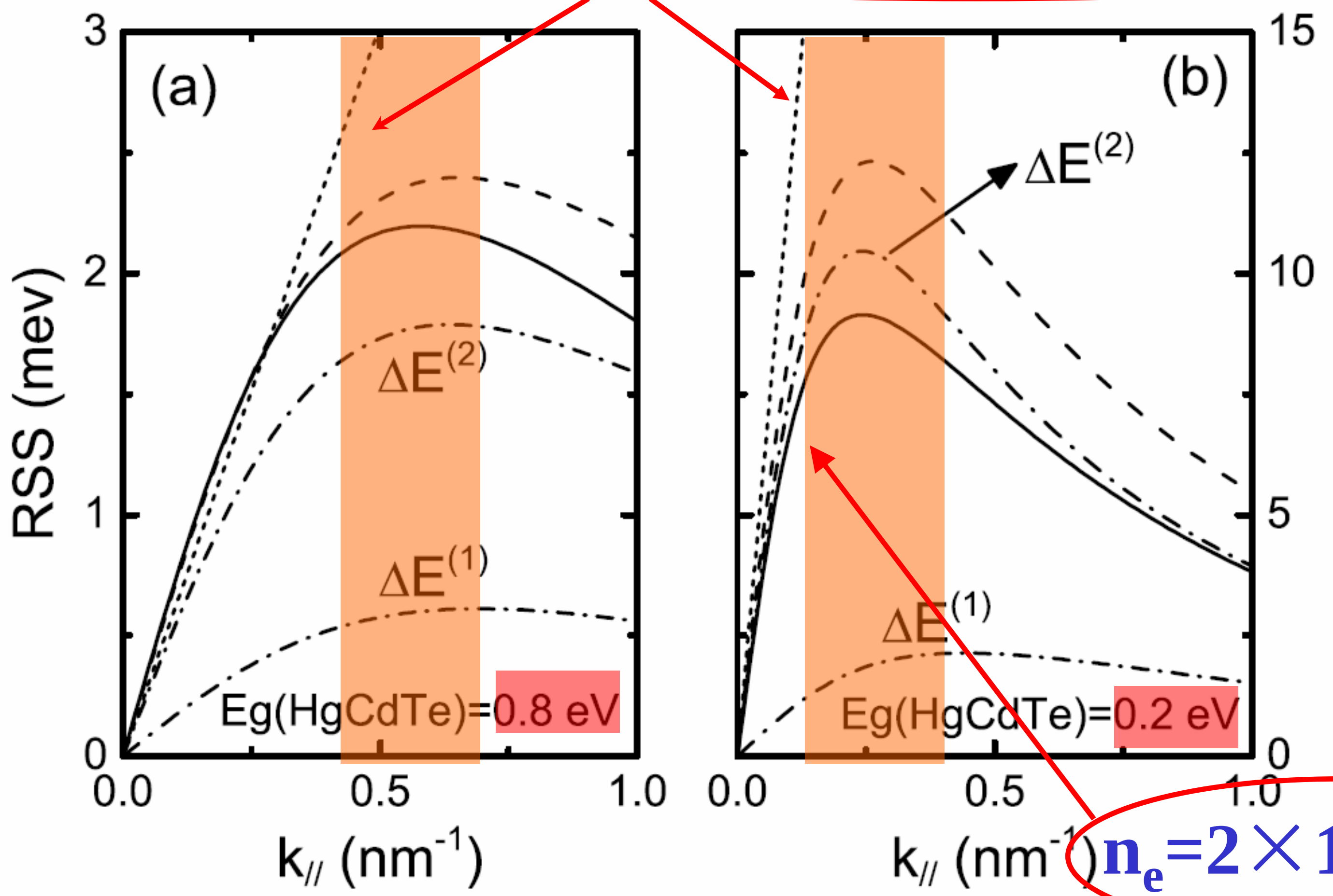
$$\Delta E_n^{(2)}(\mathbf{k}_{//}) = \frac{\hbar^2}{3m_0} E_p e F k_{//} \int dz \ |F_n(z)|^2 \left(U_{\mathrm{lh}}^{-2} - U_{\mathrm{so}}^{-2}\right).$$

Interface contribution

Interband coupling

(a) HgCdTe QW

Linear Rashba model



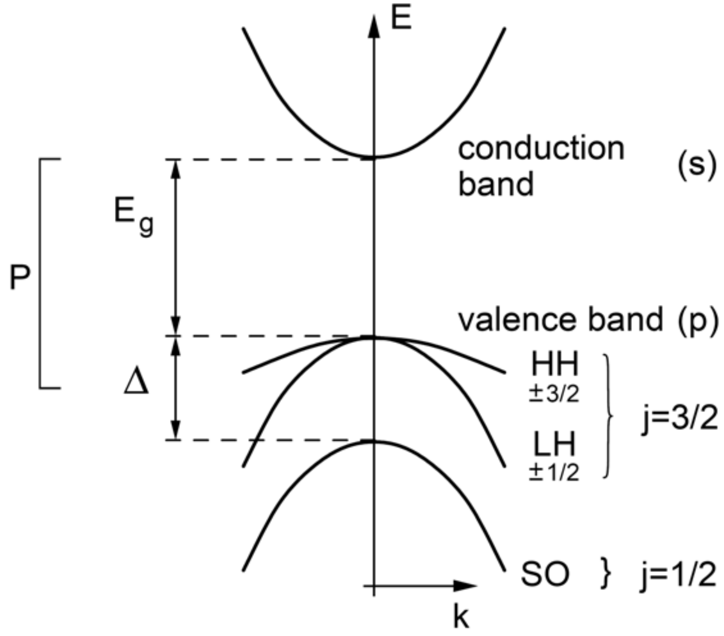
$n_e = 2\times10^{11}\,cm^{-2}$

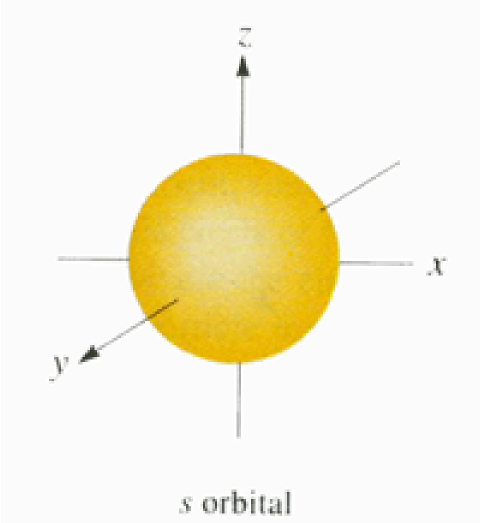
FIG. 1: RSS of the lowest subband as a function of $k_{//}$ for E_g(HgCdTe)= (a) 0.8 and (b) 0.2 eV. Solid lines denote the exact results, dashed lines are obtained from Eqs. (3) and (5)

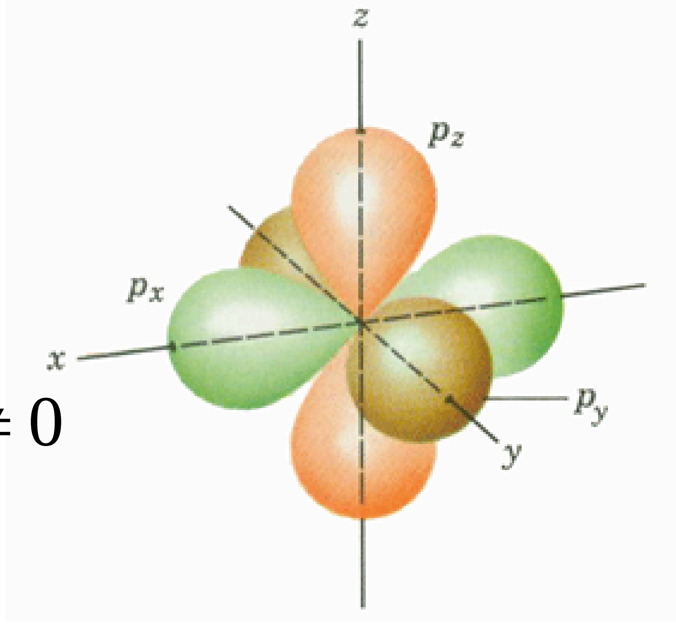
Origin of Rashba spin splitting

$$H_{SO} = \frac{\hbar}{4m^2c^2}(\nabla V \times p)\cdot\sigma$$

For s-like conduction band $\langle H_{SO} \rangle_S = 0$







For p-like conduction band $\langle H_{SO} \rangle_P \neq 0$

Origin of Rashba spin splitting

Large Δ

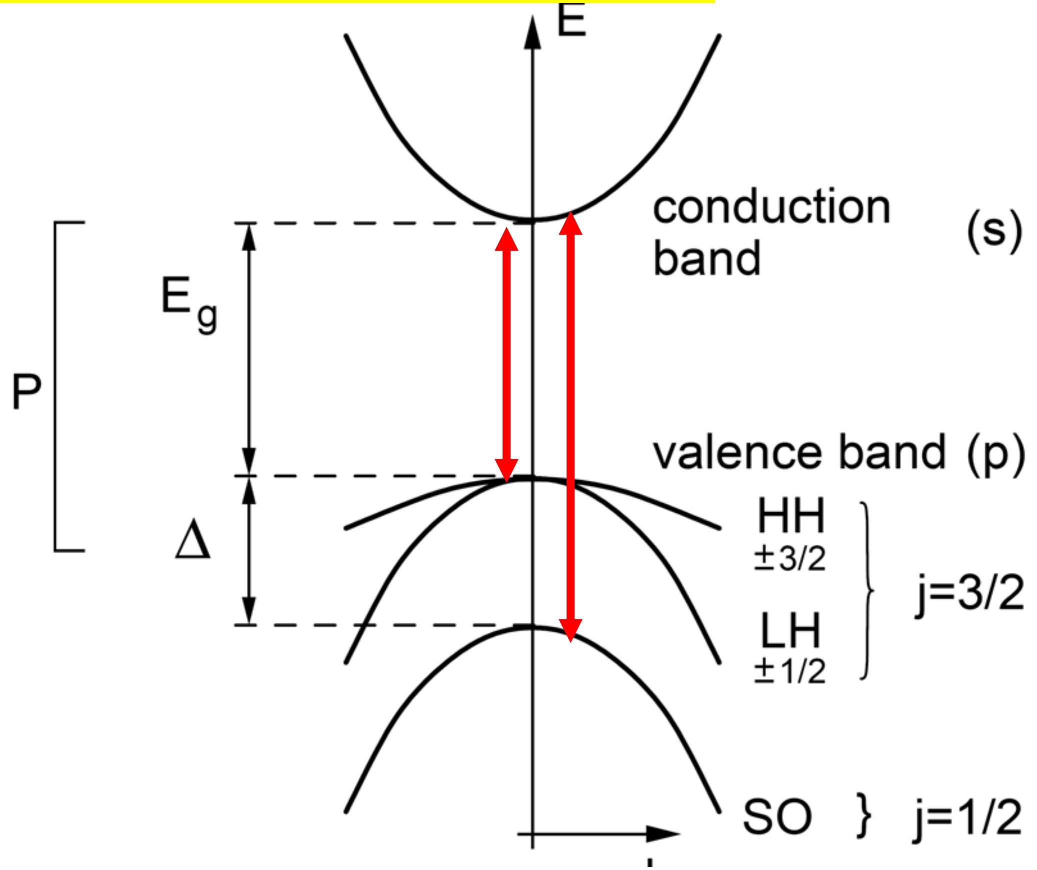
Heavy atom

\+

Interband coupling

Narrow bandgap

Large Rashba α

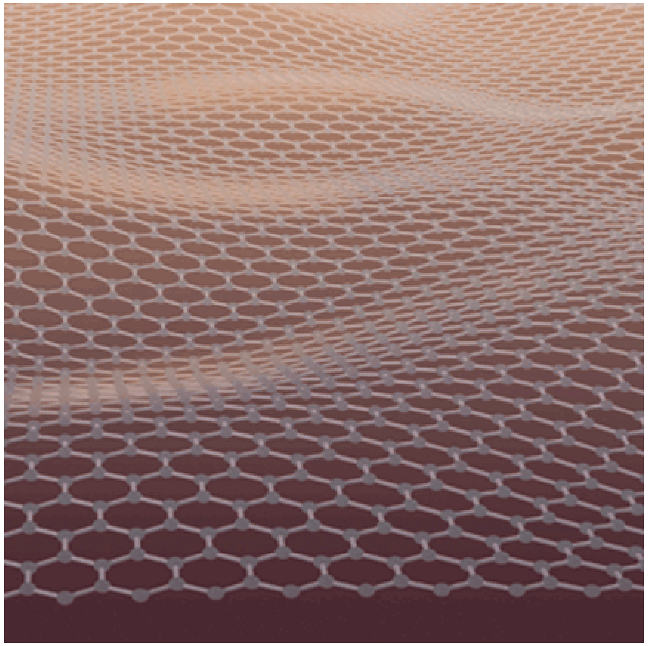


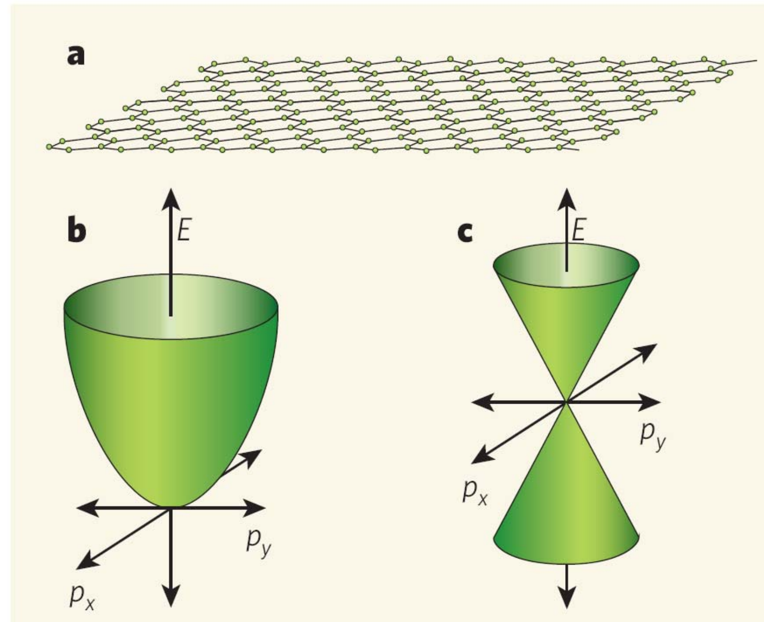
$$\Delta E_n^{(2)}(\mathbf{k}_{//}) = \frac{\hbar^2}{3m_0} E_p e F k_{//} \int dz\ \left|F_n(z)\right|^2 \left(U_{\mathrm{lh}}^{-2} - U_{\mathrm{so}}^{-2}\right).$$

The formulism can explain why large RSS in HgTe, but small in graphene!

Origin of the nonlinear behavior of RSOI!

SOI in Graphene





The spin-orbit interaction near the K point:

1, C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)(**Q**

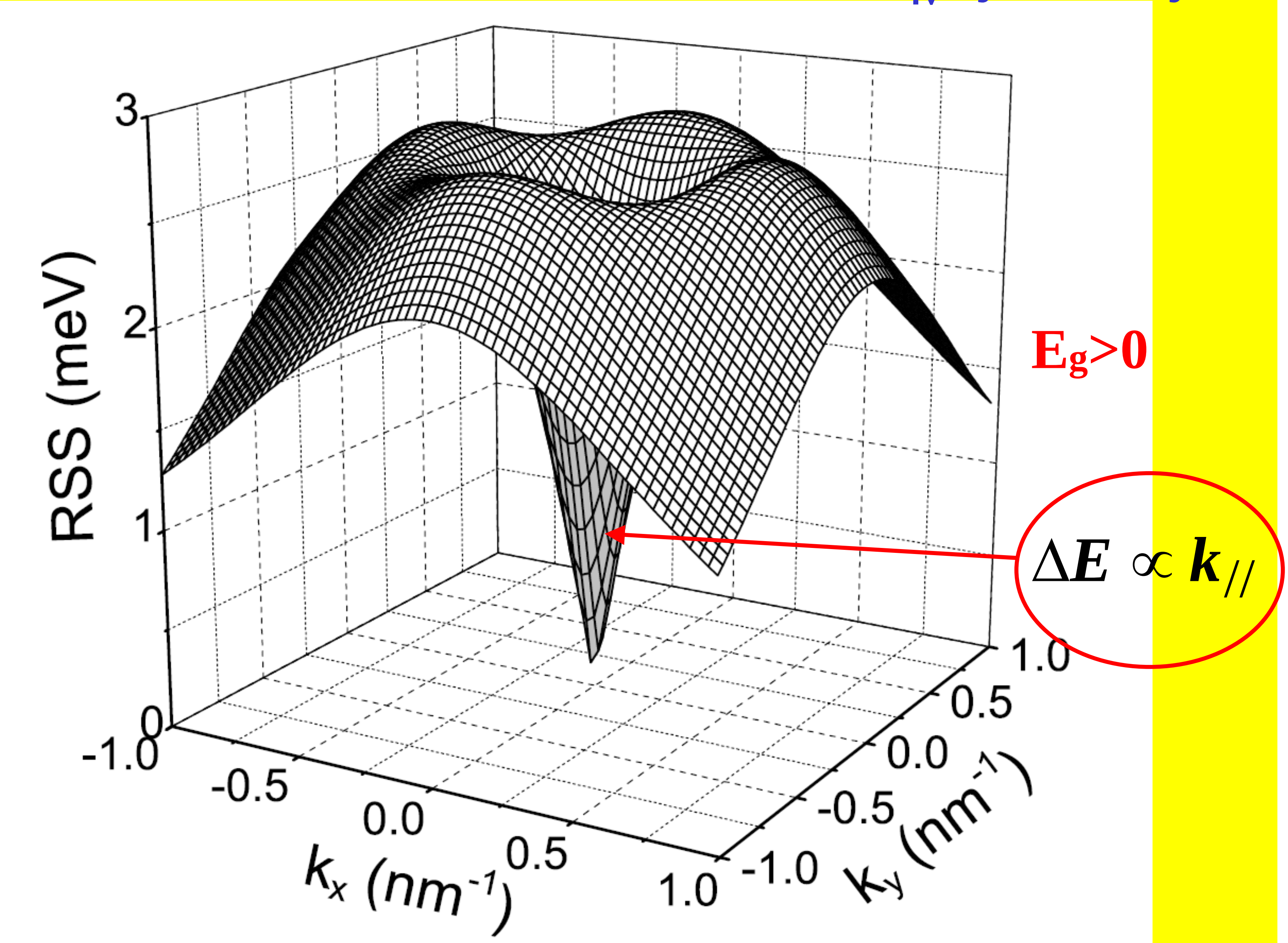
2, Phys. Rev. B 74, 165310 (2006) From MacDonald'group

λ_R ≈0.0111meV under F=50V/300nm ≈**1667kV/cm!**

SOI depends on the bandgap and the atomic mass!

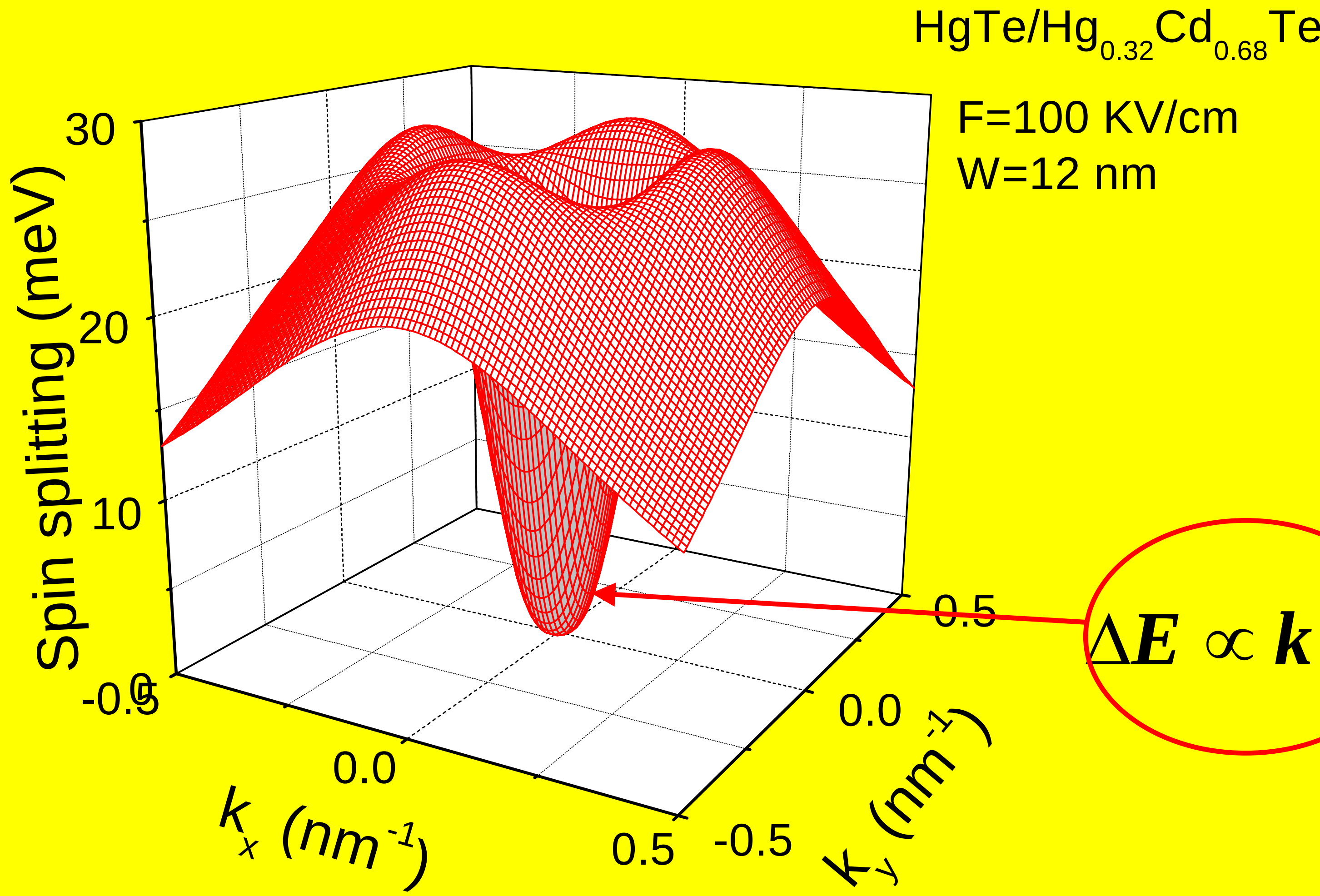
Anisotropy of Rashba spin splitting

C_{4v} symmetry



Nagative bandgap semiconductor

$E_g<0$



$\Delta E \propto k^3_{//}$

Hole-like feature

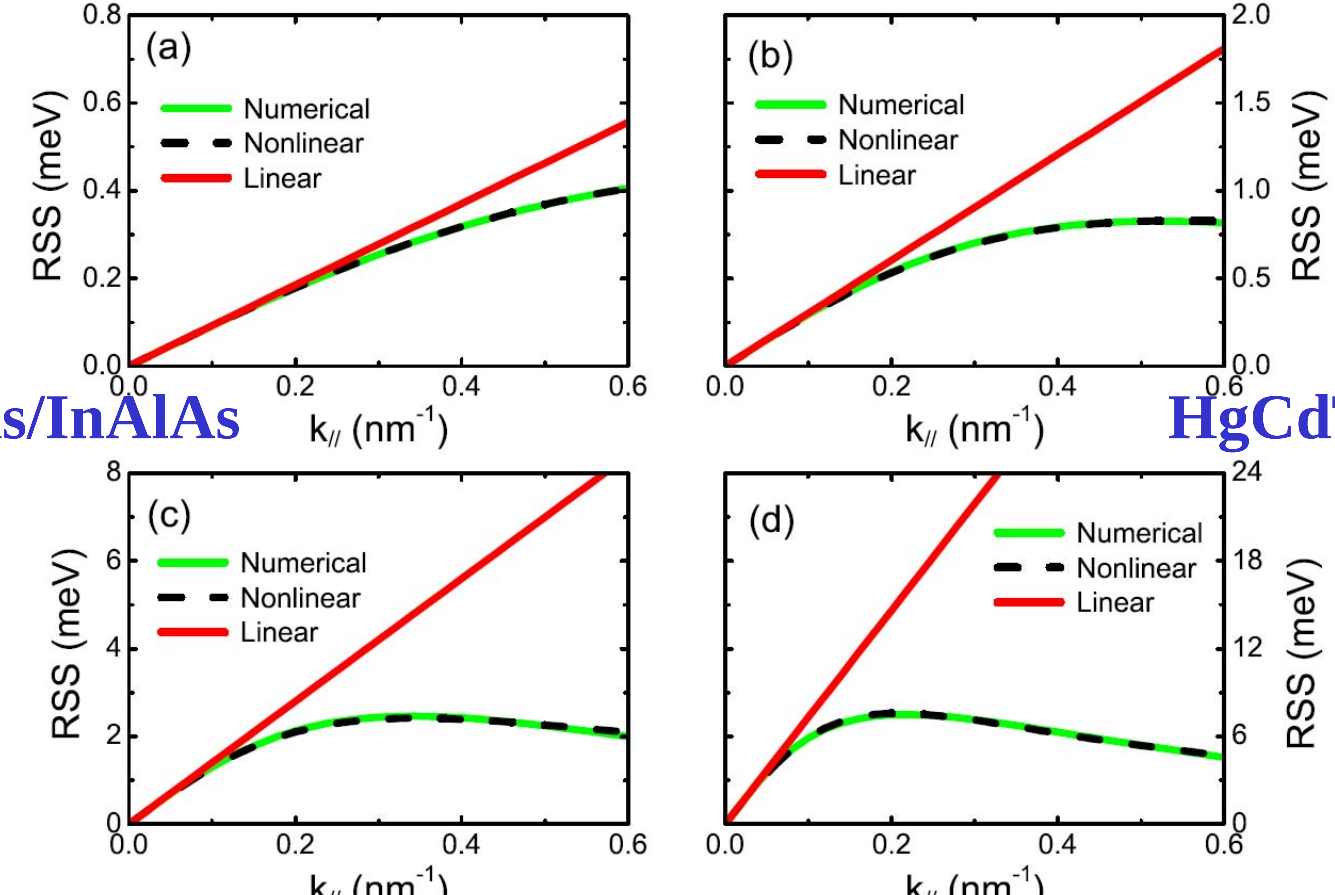
Two-coefficient nonlinear Rashba model

$$\Delta E = 2\alpha k_{//}$$

$$\Delta E = \frac{2\alpha k_{//}}{1+\beta k_{//}^2}$$

GaAs/AlGaAs | **InGaAs/GaAs**

InGaAs/InAlAs | **HgCdTe/CdTe**



W. Yang and Kai Chang, Phys. Rev. B 74, 193314(2006).

- **Many-body effect**

$$H\Psi(z) = E\Psi(z)$$

$$H = H_K + V_{conf}(z) + V_H(z)$$

$$\langle \mathcal{E}_z^{\mathrm{ext}} \rangle = \frac{e}{\epsilon\epsilon_0}\left[N_{\mathrm{A}}(z_{\mathrm{d}} - \langle z \rangle) + \frac{N_{\mathrm{s}}}{2}\right]$$

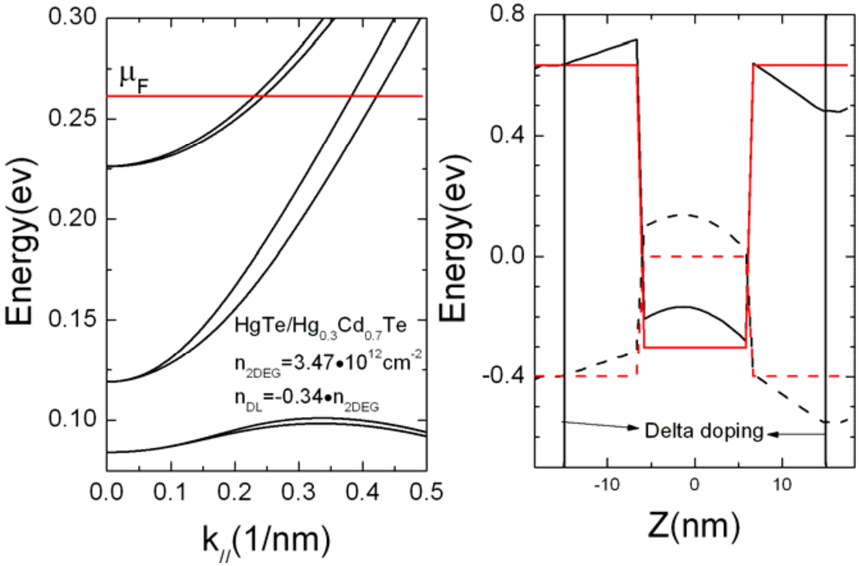
$$\mathcal{E}_z^{\mathrm{ext}}(z) = (1/e)\partial_z V_{\mathrm{H}}(z) = \frac{e}{\epsilon\epsilon_0}\left[N_{\mathrm{A}}(z_{\mathrm{d}} - z) + N_{\mathrm{s}} - \int_{-\infty}^{z} \mathrm{d}z'\, \rho(z')\right]$$

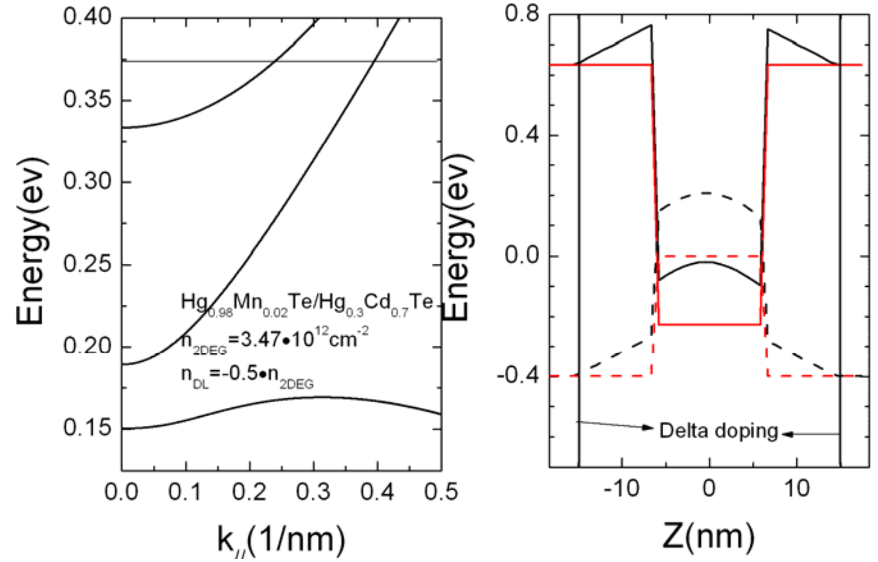
Self-consistent eight-band calculation including Hatree potential

We try to include the exchange-correlation interaction in the next step.

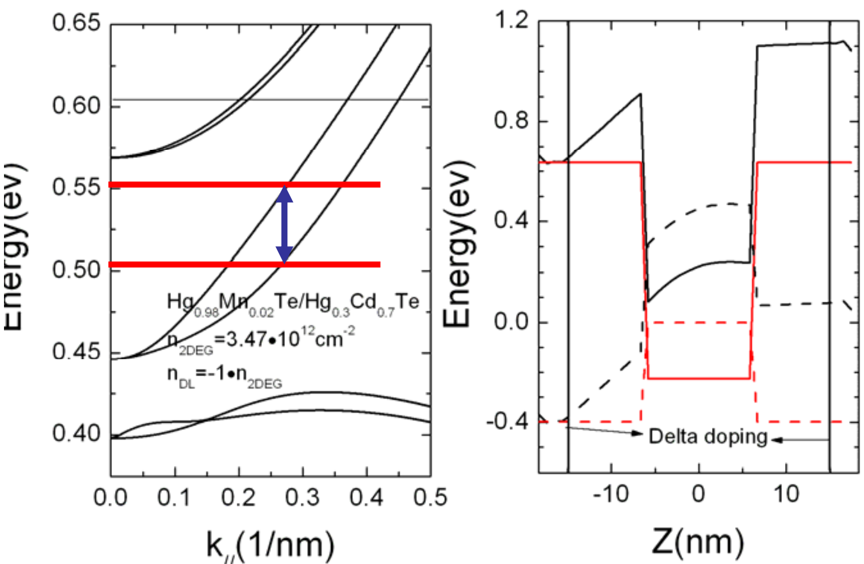
• Delta Doping In Quantum Well

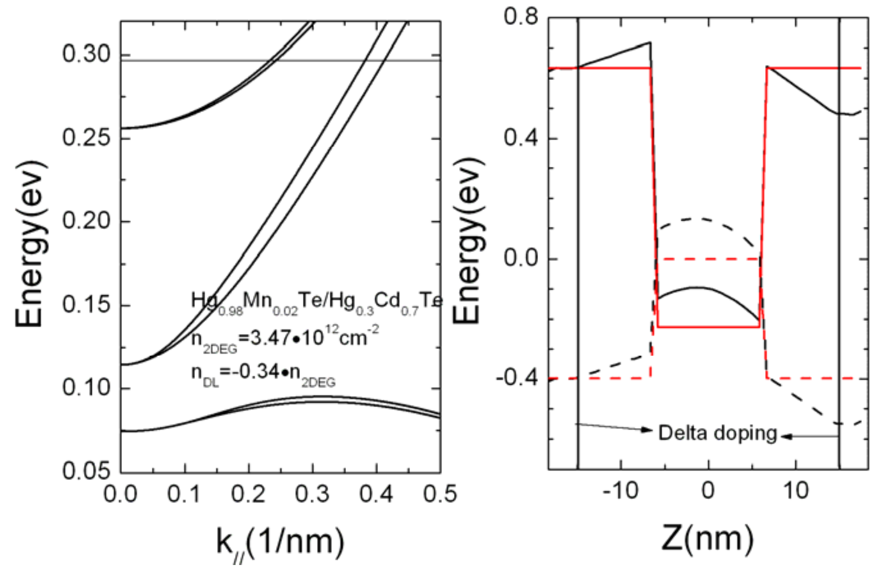
Lz=12.2nm,
Ne=3.47×10^{12}/cm^2





~50meV, Giant splitting!!! The room temp ~ 26meV



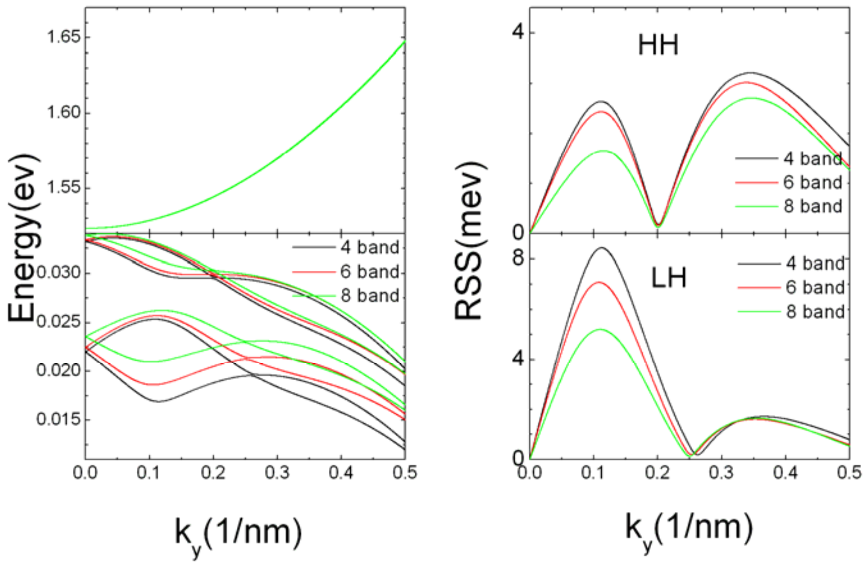


• Free standing Quantum Wire

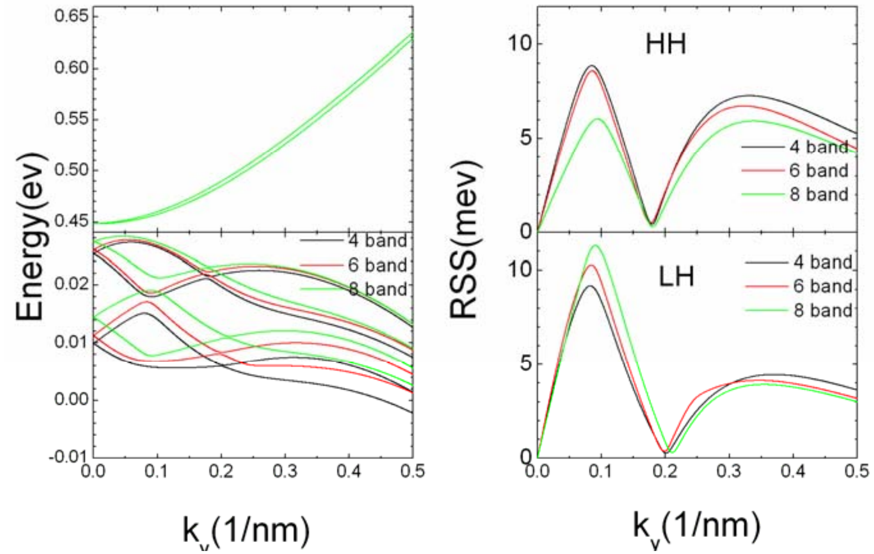
Lz=20(nm)Lx=20(nm)

E=100(kv/cm) is added in the z direction

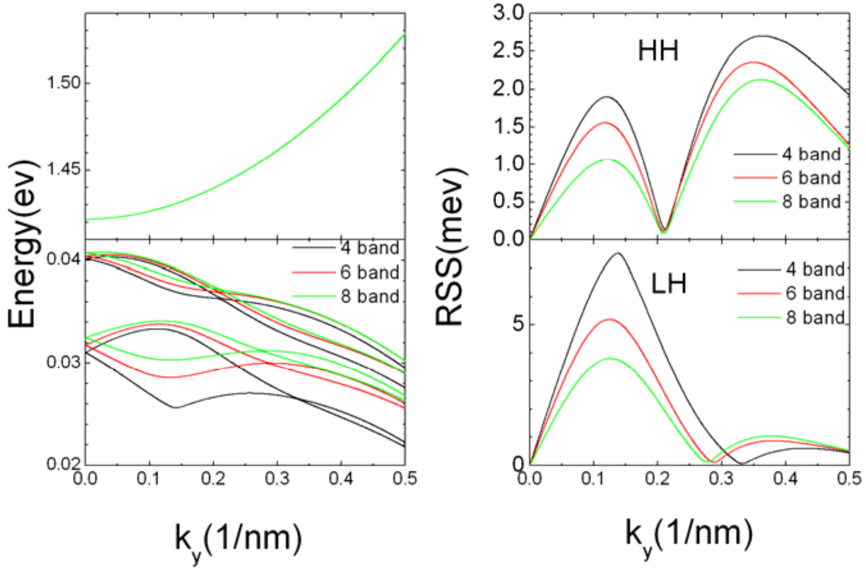
GaAs



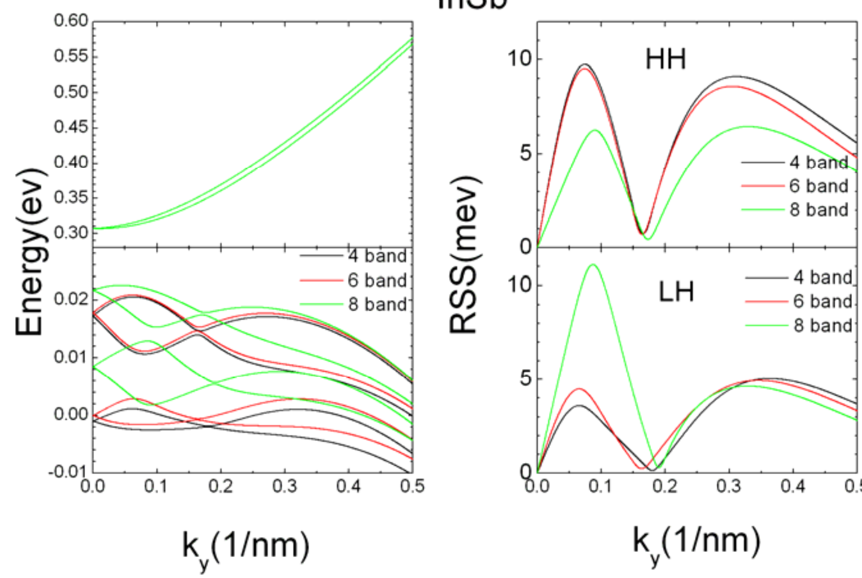
InAs



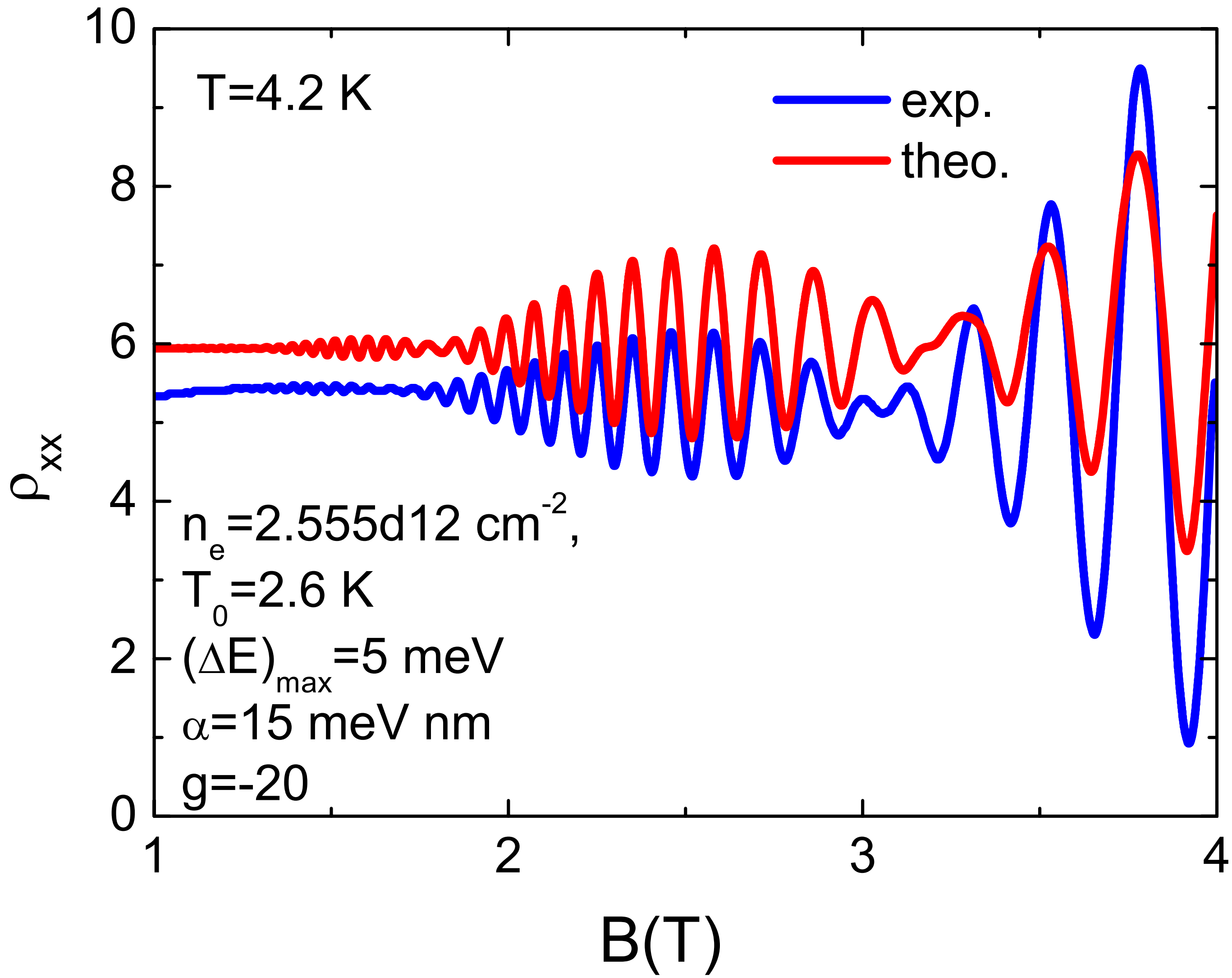
InP



InSb



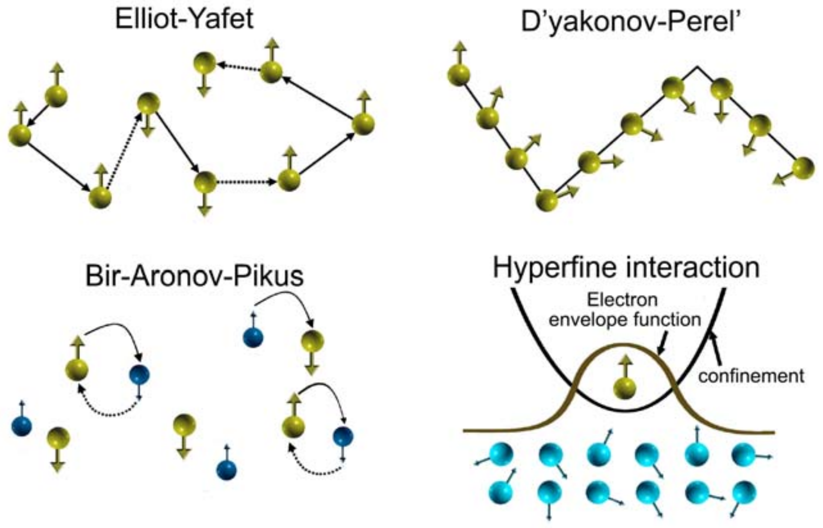
HgMnTe 2DEG (exp. Wurtzburg, Germay)



II、Spin Relaxation

Family of spin relaxation mechanisms:

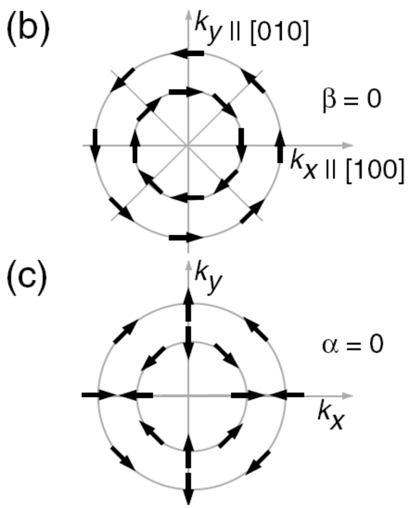
- **D’yakonov-Perel’ (DP) mechanism(SOI)**
- **Elliot-Yafet (EY) mechanism(SOI)**
- **Bir-Aronov-Pikus (BAP) mechanism(exchange)**
- **Hyperfine (s-d) mechanism(nuclear, magnetic ions)**



II、Spin Relaxation

D'yakonov Perel' (DP) mechanism:

- **Origin: spin-orbit interaction: RSOI and DSOI**



$$\mathbf{B_R} = \alpha\,(\,\mathbf{k}\times\mathbf{z}\,)$$

$$\mathcal{H}_{SO} \sim \sigma \cdot \mathbf{B}(\mathbf{k})$$

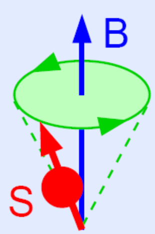
Motional narrowing

$$\tau_s \sim \tau_p^{-1}$$

Electrons ($S = 1/2$):

$$\hat{H} = H_0 + \tfrac{1}{2}\,\mathbf{S}\cdot\boldsymbol{\mathcal{B}}$$

$$\frac{d\mathbf{S}}{dt} = \frac{i}{\hbar}[\hat{H}, \mathbf{S}]$$



$$\langle\dot{\mathbf{S}}\rangle = \tfrac{1}{\hbar}\,\langle\mathbf{S}\rangle \times \boldsymbol{\mathcal{B}}$$

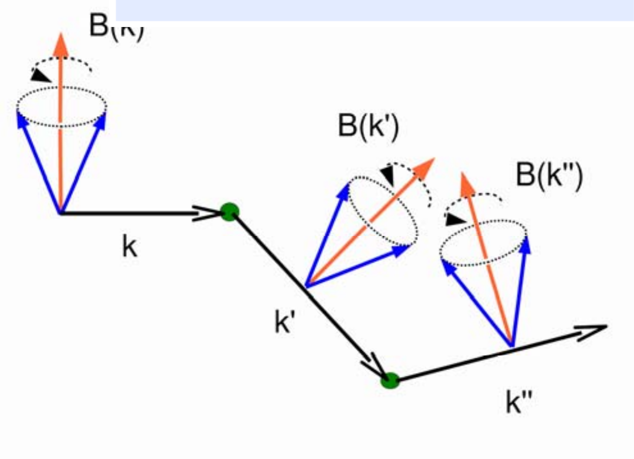
$$\frac{d\,|\langle\mathbf{S}\rangle|}{dt} = 0$$

$\langle\mathbf{S}\rangle$ = Bloch vector

Scattering effect:

1, change the in-plane momentum *k* of electron

2, change the effective magnetic field



II、Spin Relaxation

Equation of motion given by *Liouville equation*:

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho],$$

Density matrix:

$$\varrho(\mathbf{k}) := \begin{pmatrix} \varrho_{\uparrow\uparrow}(\mathbf{k}) & \varrho_{\uparrow\downarrow}(\mathbf{k}) \\ \varrho_{\downarrow\uparrow}(\mathbf{k}) & \varrho_{\downarrow\downarrow}(\mathbf{k}) \end{pmatrix}$$

Decay of the components:

1, diagonal elements (occupation number) T_1;

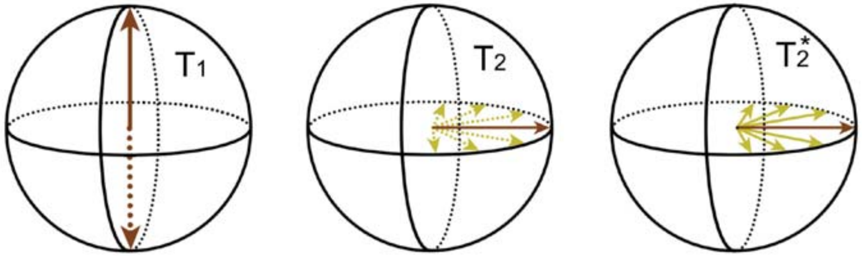
2, off-diagonal elements (decoherence) T_2

J. Kainz, U. Rössler and R. Winkler, Phys. Rev. B 70, 195322 (2004); N. S. Averkiev and L. E. Golub, Phys. Rev. B 60, 15582 (1999)

$H=H_0+H_{im}+H'$

$H'=H_{SO}$

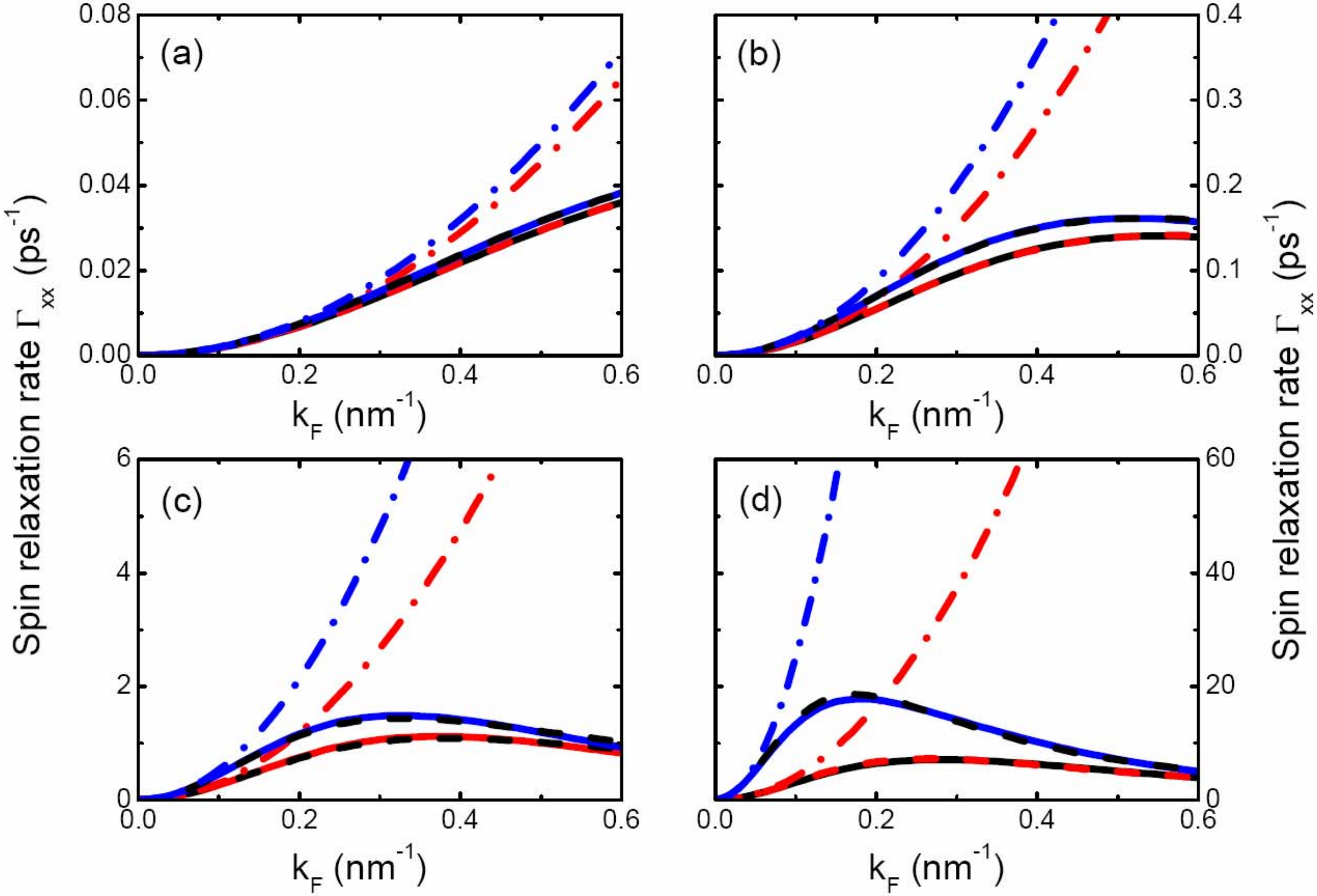
scattering



II、Spin Relaxation

Comparing between widely used linear Rashba model

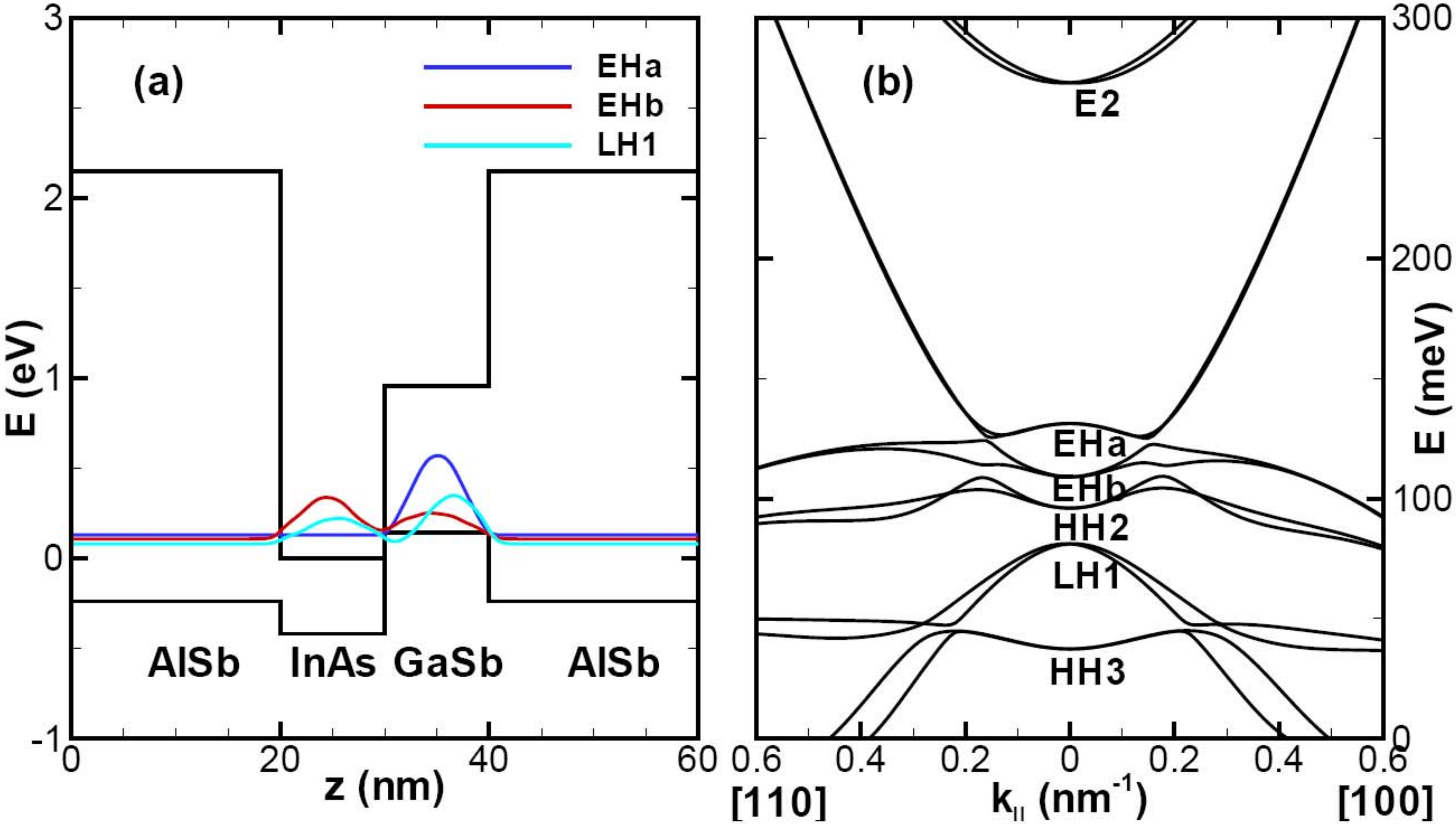
And our nonlinear Rashba model



W. Yang and Kai Chang, Phys. Rev. B 74, 193314(2006).

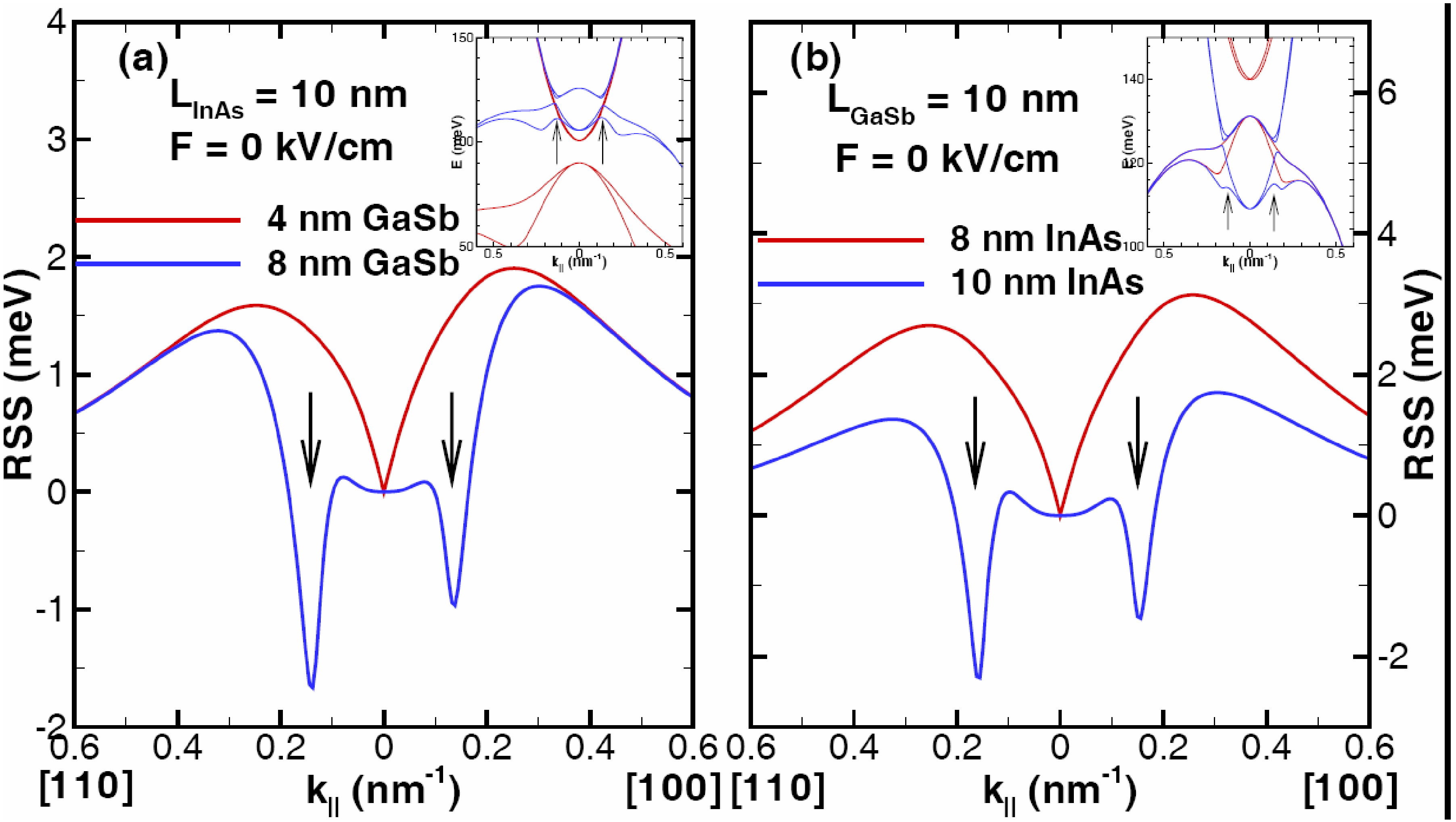
Anomalous SOI in InAs/GaSb QW

obtained from self-consistent calculation



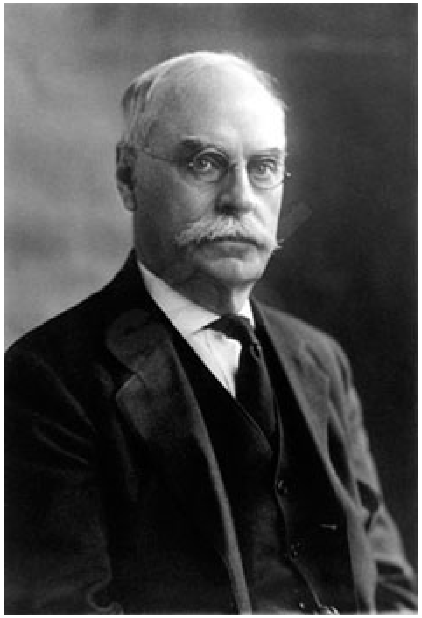
J. Li, Kai Chang, APL (in press)

Anomalous SOI in InAs/GaSb QW



Conclusion:

Nonlinear Rashba model can give correct behavior at large in-plane momentum. Nonlinear behavior of RSS is universal phenomenon in semiconductor nanostructures; can lead to surprising consequences, e.g., spin relaxation.



E. H. Hall, (1855-1938)

The Hall Effect

Classic Hall Effect (1879);

Anomalous Hall Effect (1881);

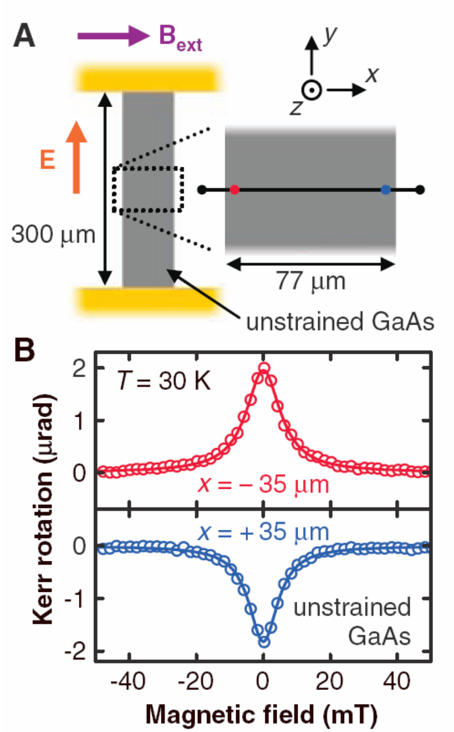
Quantum Hall Effect(1978;1982);

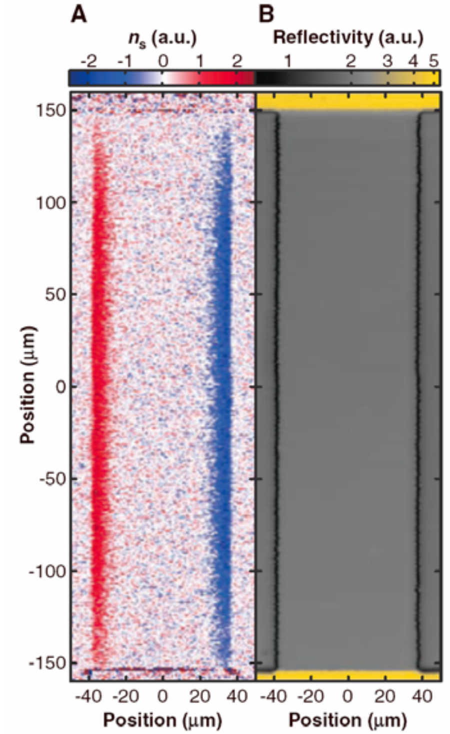
Spin Hall Effect (1971, 2004).

What is the spin Hall effect?

Electric field induces transverse spin current due to spin-orbit coupling

Spin Hall effect





Extrinsic Spin Hall Effect: impurity scattering

D'yakonov and Perel' (JETP 1971);

Hirsch (PRL 1999), Zhang (PRL 2000)

Intrinsic Spin Hall Effect: band effect (SOI)

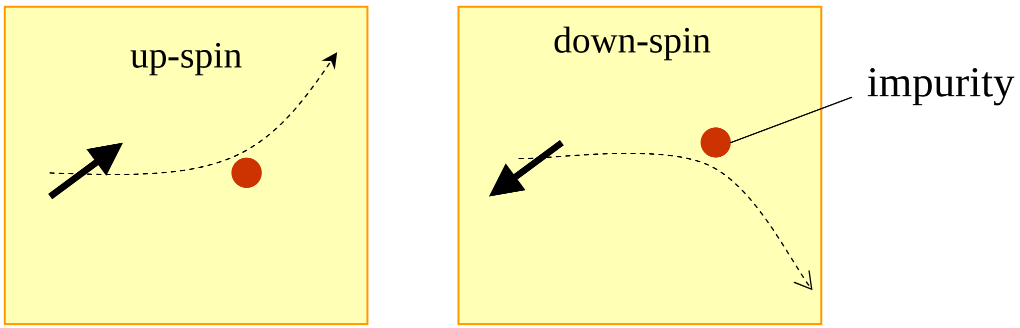
Murakami, Nagaosa, Zhang, Science (2003);

J. Sinova et al (PRL 2004)

The Extrinsic Spin Hall effect

(arising from impurity scattering with spin-orbit coupling)

impurity scattering = skew scattering + side-jump effect



skew scattering

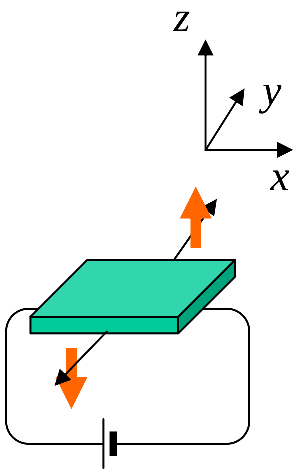
The Intrinsic Spin Hall Effect

Berry phase in momentum space
Independent of impurities

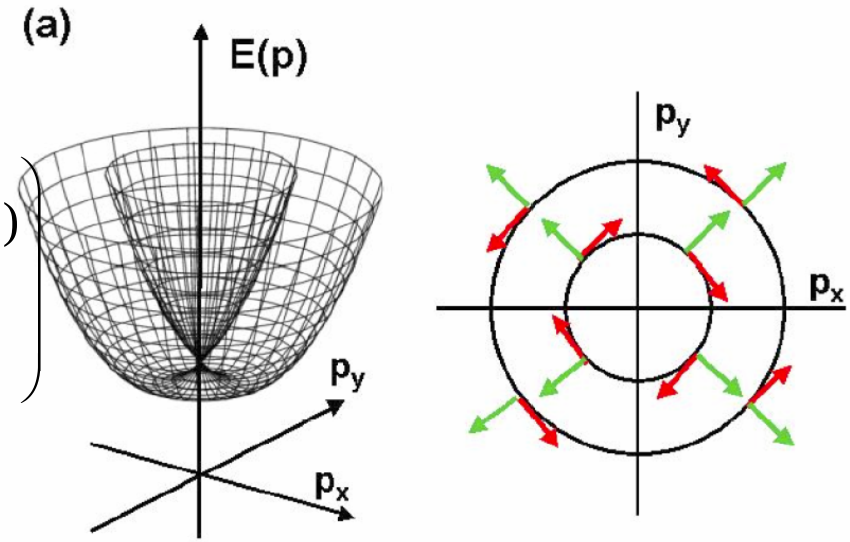
The intrinsic Spin Hall effect

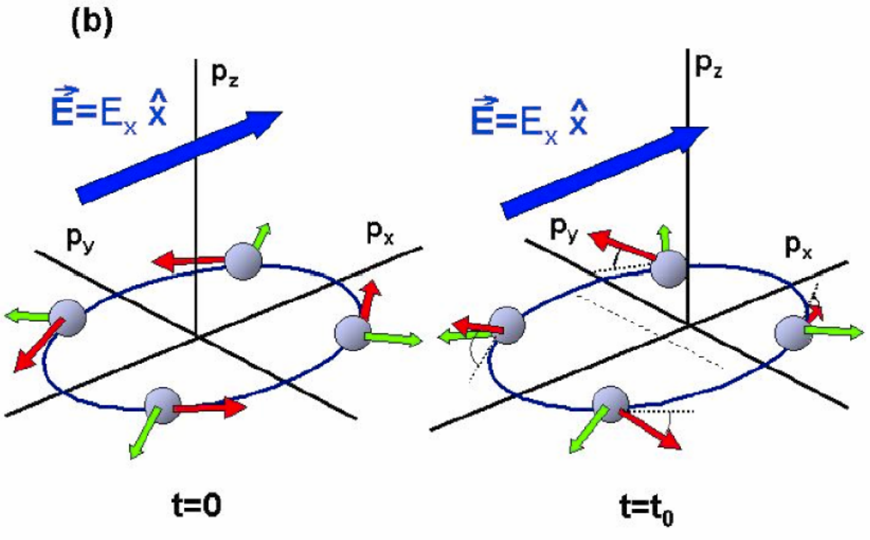
In clean 2DEG

$$H = \frac{k^2}{2m} + \lambda\left(\vec{\sigma}\times\vec{k}\right)_z = \begin{pmatrix} \frac{k^2}{2m} & \lambda(k_y + ik_x) \\ \lambda(k_y - ik_x) & \frac{k^2}{2m} \end{pmatrix}$$



2D heterostructure





PRL(2003) J.Sinova, D.Culcer, Q. Niu, A. H. MacDonald

Spin Hall effect

Kubo linear response theory:

$$\sigma_{xy}^{\mathrm{sH}}(\omega) = \frac{e\hbar}{V} \sum_{\mathbf{k}, n \neq n'} (f_{n',k} - f_{n,k}) \times \frac{\mathrm{Im}[\langle n'k | \hat{j}^{z}_{\mathrm{spin}\,x} | nk \rangle \langle nk | v_y | n'k \rangle]}{(E_{nk} - E_{n'k})(E_{nk} - E_{n'k} - \hbar\omega - i\eta)}$$

Linear Rashba Model: (PRL(2004))

$$\sigma_{\mathrm{sH}} \equiv -\frac{j_{s,y}}{E_x} = \frac{e}{8\pi}, \quad n_{2\mathrm{D}} > n^*_{2\mathrm{D}}$$

$$\sigma_{\mathrm{sH}} = \frac{e}{8\pi} \frac{n_{2\mathrm{D}}}{n^*_{2\mathrm{D}}}. \qquad n_{2\mathrm{D}} < n^*_{2\mathrm{D}}$$

$$n^*_{2\mathrm{D}} \equiv m^2 \lambda^2 / \pi \hbar^4$$

Suppression of the persistent spin Hall current by defect scattering

J. Inoue, G. E. Bauer and L. W. Molenkamp PRB (2004)

1, single parabolic band,

2, linear response theory,

3, linear Rashba model,

4, short-range potential, Born approximation.

Conclusions:

1, vanishing spin Hall effect including the vertex correction

2, dominant forward scattering leads to a nonvanishing SHE.

争论和分歧

•**Sinova**等人**[PRL, 2003]** 得出本征自旋霍尔效应为普适常数这一结论时，没有考虑杂质散射效应。

• **Inoue**等人**[PRB, 2004]** 考虑δ型杂质散射(即动量无关散射) 对速度算符的顶角修正后，指出即使杂质散射极弱，本征自旋霍尔效应也会被完全抵消，而且不会有自旋积累。同时，**Inoue** 等人认为当杂质散射与动量有关(即不是严格δ势) 时，本征自旋霍尔效应不会被完全抵消。

• **S. Murakami**等人**[PRL (2004)]**发现即便考虑杂质散射，量子阱中空穴的自旋霍尔电导依然存在。

• **Raimondi**等人**[PRB, 2005]** 认为即使杂质散射与动量有关，本征自旋霍尔效应仍然为零

• **Dimitrova [PRB,2005]** 经计算得到对任何形式的**Rashba** 劈裂，以及任何形式的能带色散，短程杂质散射总是完全抵消本征自旋霍尔效应。

• **Krotkov**等人**[PRB, 2006]** 持类似**Inoue**的观点，他们认为线性**Rashba** 模型、抛物型能带情况下计算得到本征自旋霍尔效应为零只是一种偶然现象，当能带非抛物时，本征自旋霍尔效应不为零。

• **Khaetskii [PRL 2006]** 认为对线性**Rashba** 劈裂及抛物型能带，**Born** 近似下的本征自旋霍尔效应总是为零。

SHE的存在和能带的对称性有关！

以上所有研究工作都是将电子和空穴分别独立地处理的！

Controversy：Does Intrinsic SHE vanish in 2DEG and/or 2DHG?

Conclusions：SHE vanishes at

1, Linear Rashba model,

2, parabolic band,

3, short-range impurity potential, Born approximation.

1, Nonlinear behavior of Rashba SOI

2, Non-parabolic band

→ **Narrow band gap semiconductors**

↓

Unified description of SHE based on the eight-band Kane model

PRL 100, 056602(2008)

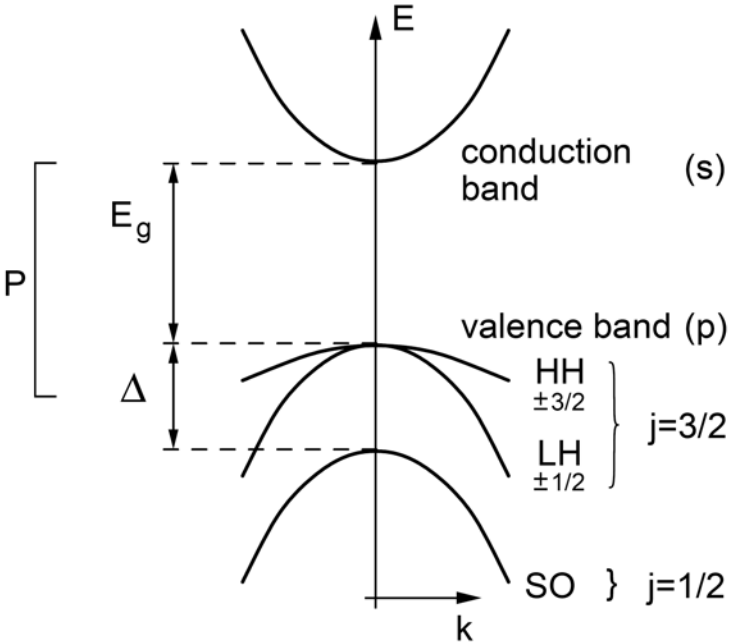
Unsolved:

1, Long-range potential? (Khaetskii 2006：SHE still vanishes!)

2, Beyond Born approxiamtion

The operators in the eight band Kane model

$$\phi_1 = S\uparrow,$$
$$\phi_2 = S\downarrow,$$
$$\phi_3 = \left|\frac{3}{2},\frac{3}{2}\right\rangle = \frac{1}{\sqrt{2}}(X+iY)\uparrow,$$
$$\phi_4 = \left|\frac{3}{2},\frac{1}{2}\right\rangle = \frac{i}{\sqrt{6}}\left[(X+iY)\downarrow -2Z\uparrow\right],$$
$$\phi_5 = \left|\frac{3}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{6}}\left[(X-iY)\uparrow +2Z\downarrow\right],$$
$$\phi_6 = \left|\frac{3}{2},-\frac{3}{2}\right\rangle = \frac{i}{\sqrt{2}}(X-iY)\downarrow,$$
$$\phi_7 = \left|\frac{1}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left[(X+iY)\downarrow +Z\uparrow\right],$$
$$\phi_8 = \left|\frac{1}{2},-\frac{1}{2}\right\rangle = -\frac{i}{\sqrt{3}}\left[(X-iY)\uparrow -Z\downarrow\right]$$



PRL 100, 056602(2008)

For the operators O:

$$\tilde{\mathbf{O}}_{j\mathbf{k},j'\mathbf{k}'} = \mathbf{O}_{j\mathbf{k},j'\mathbf{k}'} + \sum_{l\mathbf{k}''}\Big[\mathbf{O}_{j\mathbf{k},l\mathbf{k}''}(E-E_l)^{-1}(H_{kp})_{l\mathbf{k}'',j'\mathbf{k}'} + (H_{kp})_{j\mathbf{k},l\mathbf{k}''}(E-E_l)^{-1}\mathbf{O}_{l\mathbf{k}'',j'\mathbf{k}'}\Big].$$

The operators in the eight band Kane model

$$[\tilde{v}_\alpha(\mathbf{k})]_{jj'} = \frac{p^\alpha_{jj'}}{m_0} + \delta_{jj'}\frac{\hbar k_\alpha}{m_0} + \frac{\hbar}{4m_0^2c^2}(\boldsymbol{\sigma}\times\nabla V)_{\mu\mu'} + \frac{\hbar}{m_0^2}\sum_{\beta,l}\left(\frac{p^\alpha_{jl}p^\beta_{lj'}}{E-E_l}k_\beta + k_\beta\frac{p^\beta_{jl}p^\alpha_{lj'}}{E-E_l}\right).$$

二维电子气的二维直流自旋霍尔电导率 $\sigma^{\alpha\beta\gamma}_{2D} = \langle J^\alpha_\beta\rangle / E^\gamma$

$$\sigma^{2D}_{\alpha\beta\gamma} = \sigma^{3D}_{\alpha\beta\gamma}L$$

$$\sigma^{3D}_{\alpha\beta\gamma} = \frac{1}{\hbar V}\lim_{\omega\to 0}\frac{e}{i\omega}\left[G^r_{AB}(\omega) - G^r_{AB}(0)\right]$$

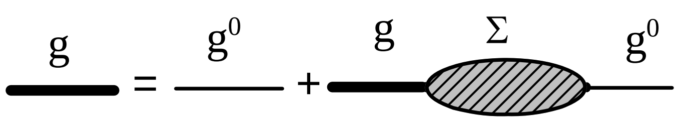
PRL 100, 056602(2008)

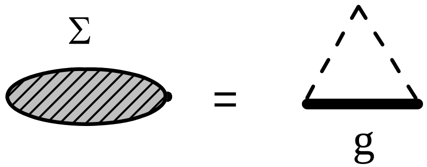
Intrinsic spin Hall conductivity σ_{sH} with vertex correction

We adopt:

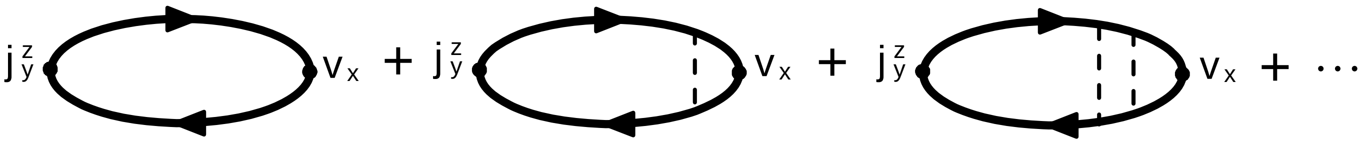
1，**8band Hamiltonian in axial approximation ($\gamma_2 = \gamma_3$)**

2，Green function: self-consistent Born approximation



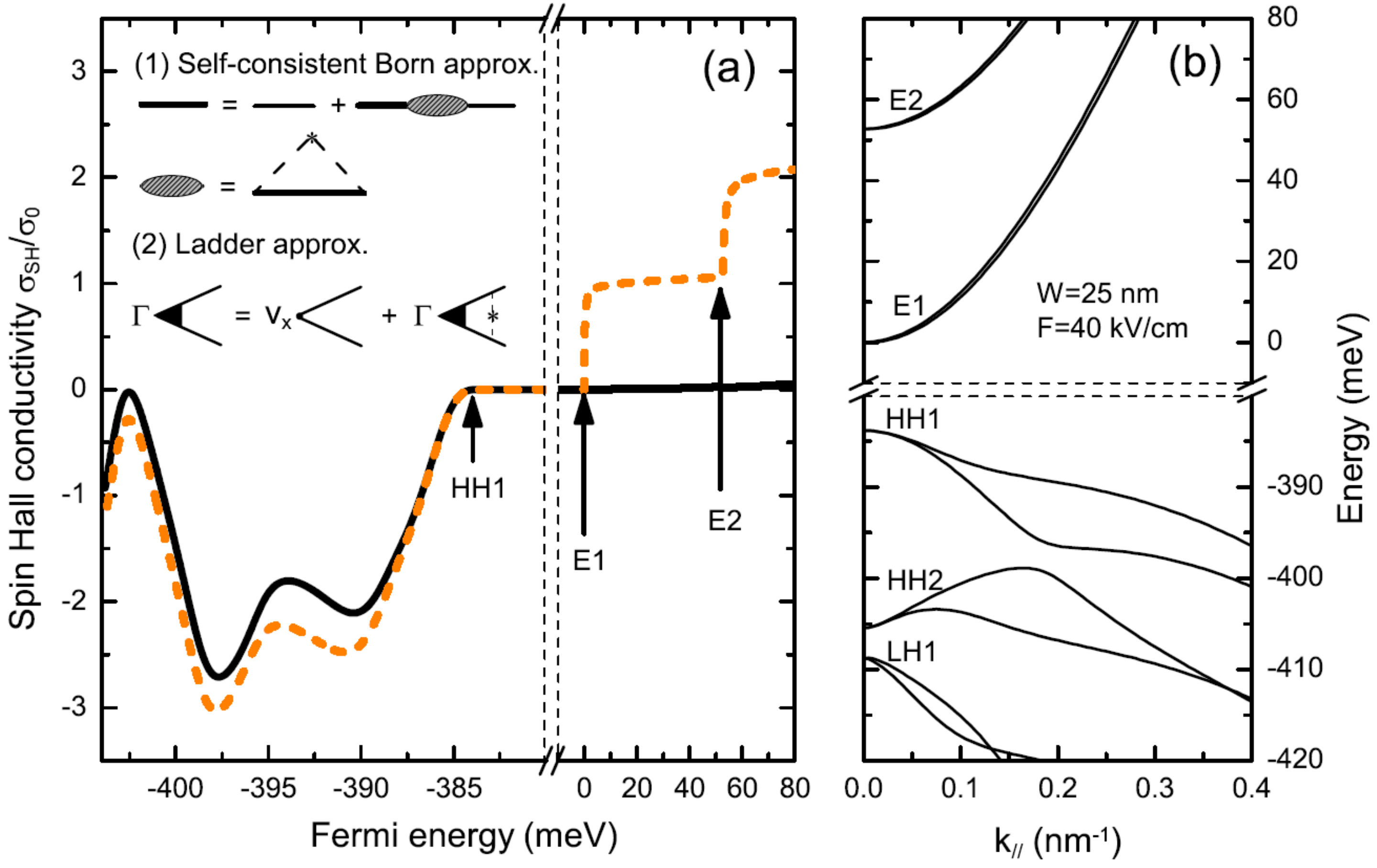


Bethe-Salpeter equation

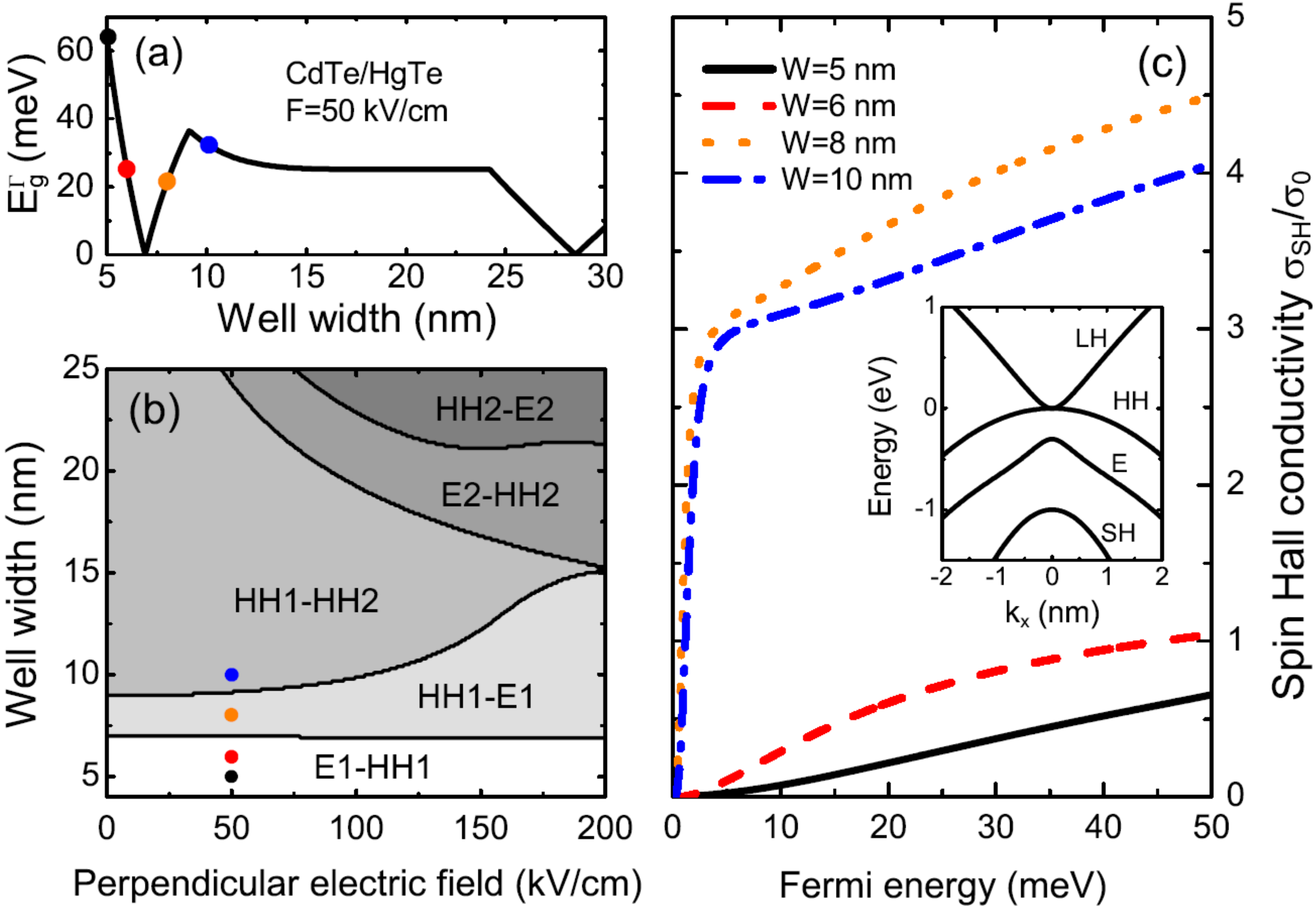


PRL 100, 056602(2008)

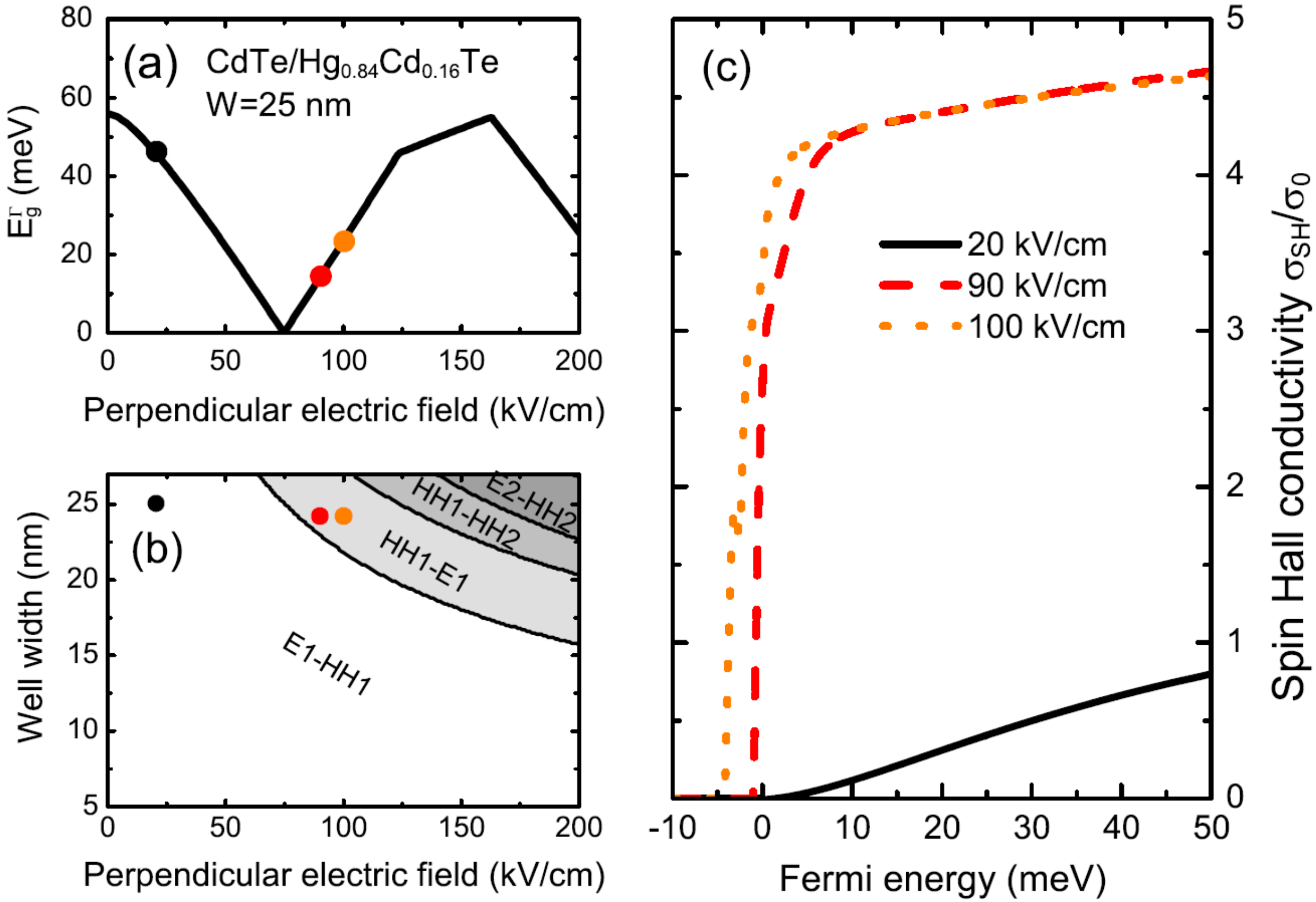
Spin Hall effect

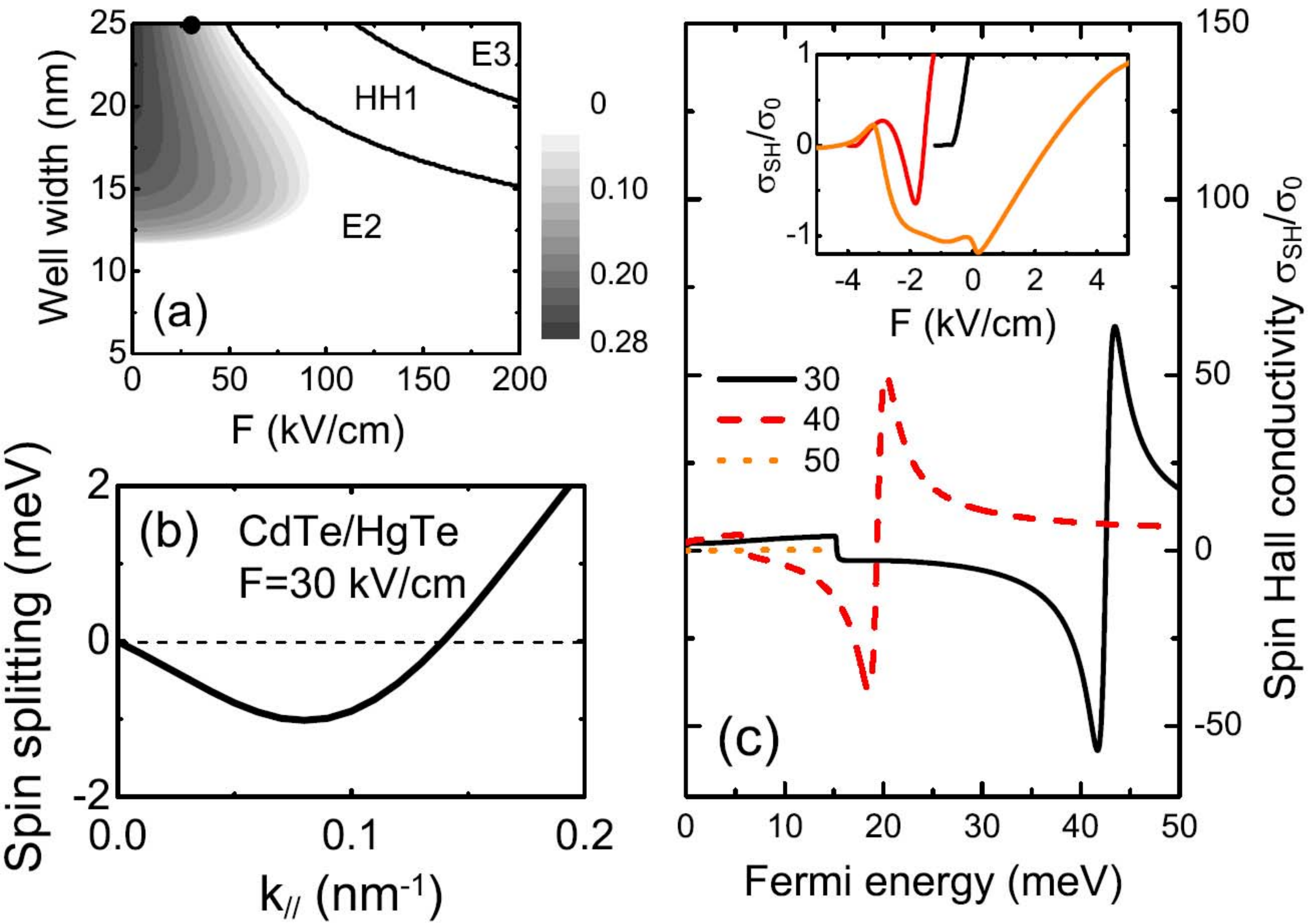


Intrinsic spin Hall conductivity σ_{sH} with vertex correction



Spin Hall effect





(a)
Well width (nm)
F (kV/cm)
E3
HH1
E2
0
0.10
0.20
0.28
(b)
CdTe/HgTe
F=30 kV/cm
Spin splitting (meV)
$k_{//}$ (nm^{-1})
(c)
σ_{SH}/σ_0
F (kV/cm)
30
40
50
Fermi energy (meV)
Spin Hall conductivity σ_{SH}/σ_0

Conclusions:

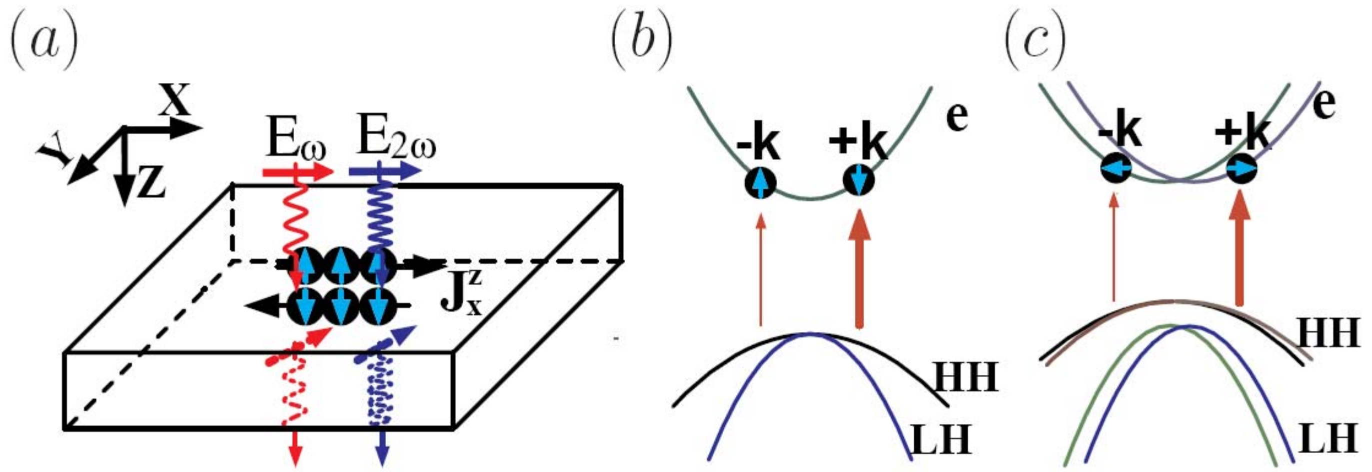
- Unified description of Spin Hall effect;
- Switching of SHE utilizing the external electric field;
- Could provides us a possible way to distinguish extrinsic and intrinsic Spin Hall effects.

Optical detection of spin current

Pure spin current: the spin-up and spin-down electron currents have equal magnitudes but travel in opposite directions
→ vanishing charge current and total spin
→ No conventional magneto-optical effects

Asymmetric distribution in k space
$f(-k)=f(+k)\neq 0$

Transition rate of QUIC:
$W(-k)=0,\ W(+k)\neq 0$



Theory:

The Luttinger-Kohn Hamiltonian:

$$H_h(k,\theta,\varphi) = \frac{\hbar^2 k^2}{2m_0} \begin{pmatrix} H_h & L & M & 0 \\ L^* & H_l & 0 & M \\ M^* & 0 & H_l & -L \\ 0 & M^* & -L^* & H_h \end{pmatrix} - \frac{\hbar^2 k^2}{2m_0}\gamma_1$$

where

$$H_h = -\gamma_2 \sin^2\theta + 2\gamma_2 \cos^2\theta$$

$$H_l = +\gamma_2 \sin^2\theta - 2\gamma_2 \cos^2\theta = -H_h$$

$$M = -\sqrt{3}\gamma_2 \sin^2\theta e^{-2i\varphi}$$

$$L = i2\sqrt{3}\gamma_2 \cos\theta \sin\theta e^{-i\varphi}$$

The Valkov-type solution

$$\psi_{c,v}(\mathbf{k},r,t) = u_{c,v}(\mathbf{k},r)\exp[i\mathbf{k}\bullet\mathbf{r} - i\omega_{c,v}(\mathbf{k})t + \frac{ie}{m_{c,v}}\int_0^t \mathbf{k}\bullet A(\tau)d\tau]$$

The transition rate is calculated using Fermi golden rule:

$$S = -\frac{i}{\hbar}\int_{-\infty}^{\infty} dt' \int d^3 r \psi *_c (\mathbf{k}, r, t') \frac{-e}{m_0 c} A \bullet P \psi_v (\mathbf{k}', r, t')$$

The transition rate

$$w(\mathbf{k}) = \lim_{t\to\infty} \frac{d}{dt} |S|^2$$

$$= \{ \left(\frac{\eta_1}{2}\right)^2 |\mathbf{p}_{vc} \bullet \mathbf{a}_1|^2 A_1^2 + |\mathbf{p}_{vc} \bullet \mathbf{a}_2|^2 A_2^2 + A_1 A_2 \frac{\eta_1}{2}$$

$$\left[(\mathbf{p}_{vc} \bullet \mathbf{a}_1)^* (\mathbf{p}_{vc} \bullet \mathbf{a}_2) \exp i(2\varphi_1 - \varphi_2) + (\mathbf{p}_{vc} \bullet \mathbf{a}_1)(\mathbf{p}_{vc} \bullet \mathbf{a}_2)^* \exp i(-2\varphi_1 + \varphi_2) \right]$$

where

$$\eta_1 = \frac{eA_1}{\omega c m_{cv}} \mathbf{k} \bullet \vec{a}_1, \frac{1}{m_{cv}} = \frac{1}{m_c} - \frac{1}{m_v}$$

Consider ω and 2ω beams are polarized along the x direction, the electric fields are given by

$$\mathbf{E}(\omega) = E(\omega) e^{i\varphi_1} \hat{x}$$

$$\mathbf{E}(2\omega) = E(2\omega) e^{i\varphi_2} \hat{x} = E(2\omega) e^{i\varphi_2} \left[(\hat{x} + i\hat{y}) + (\hat{x} - i\hat{y}) \right]$$

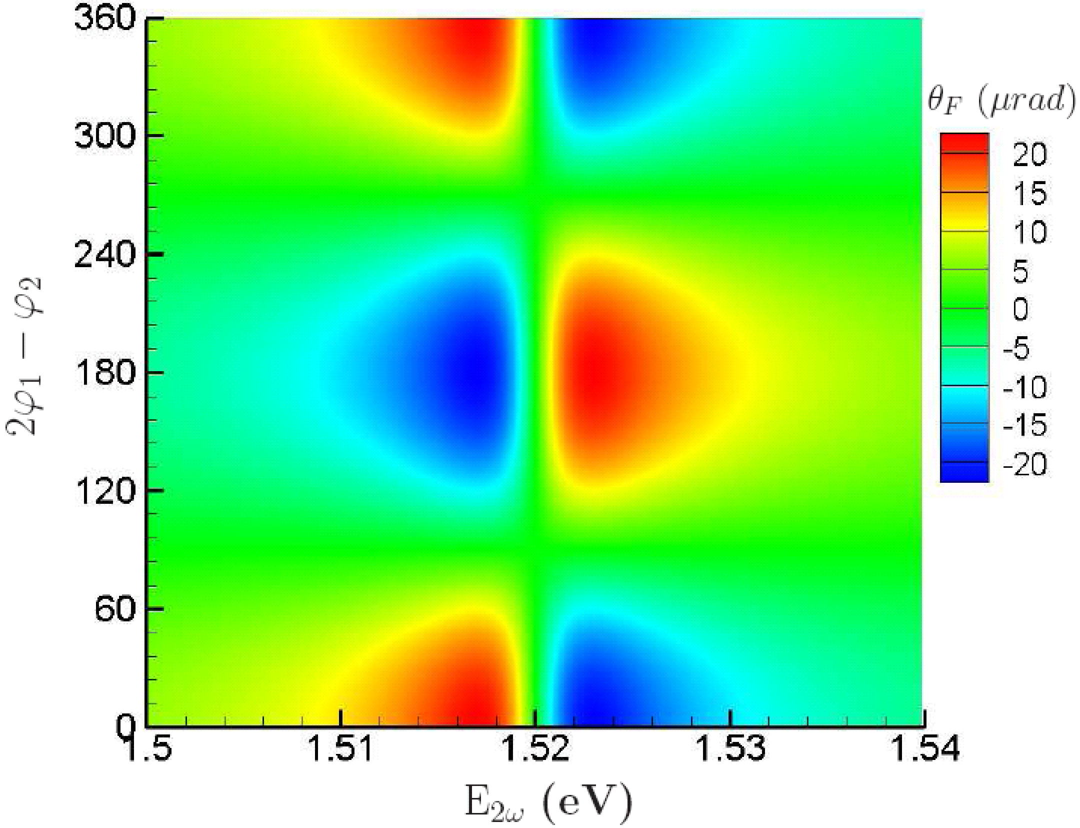
Magneto-optical Effects: Faraday Rotation

$$\theta_F(\omega) = \frac{\omega}{c} Re(N_+ - N_+)$$

$$\begin{aligned} N_+ - N_- \propto W_+(+k) + W_+(-k) - W_-(+k) - W_-(-k) \\ = C_0[C_{i1}(f_u + f_d)\cos(2\varphi_1 - \varphi_2) + C_{i2}\sin(2\varphi_1 - \varphi_2) \\ \times \sin(2\varphi_e)(f_d - f_u) + C_l(f_u - f_d)], \end{aligned}$$

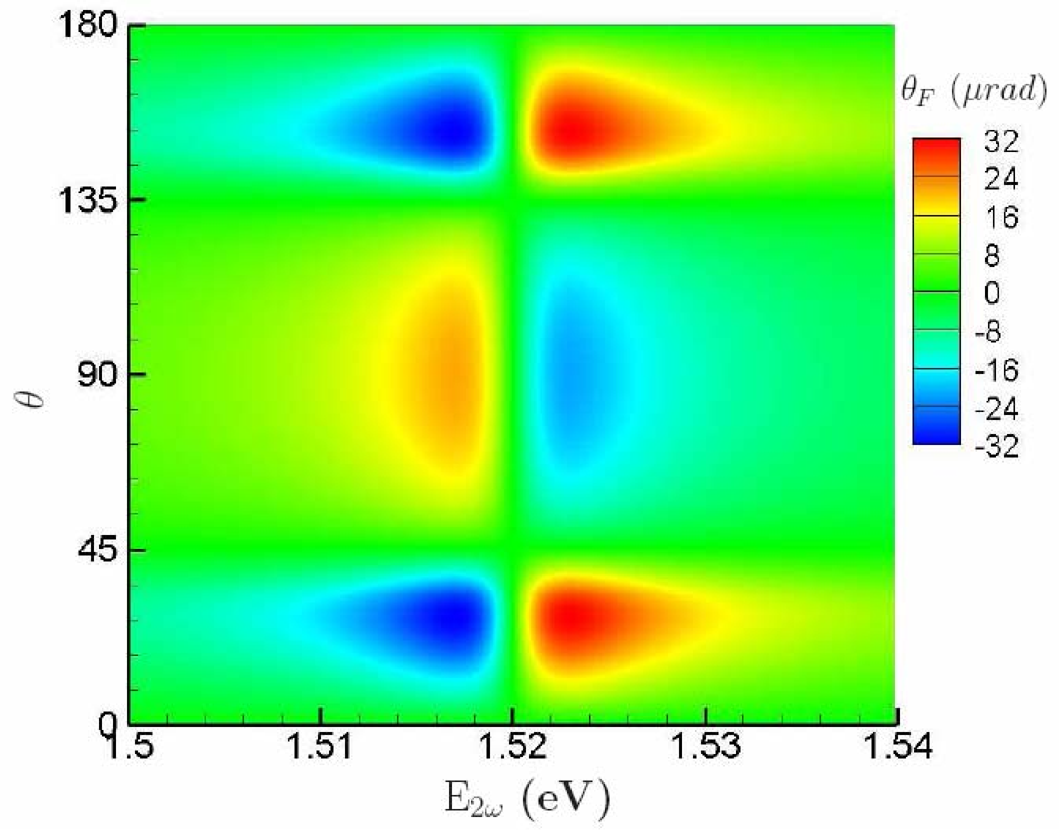
If fd=fu (Pure spin current)
→ Conventional Magneto-optical Effect vanishes,
but FR of QUIP appears

Results and Discussion



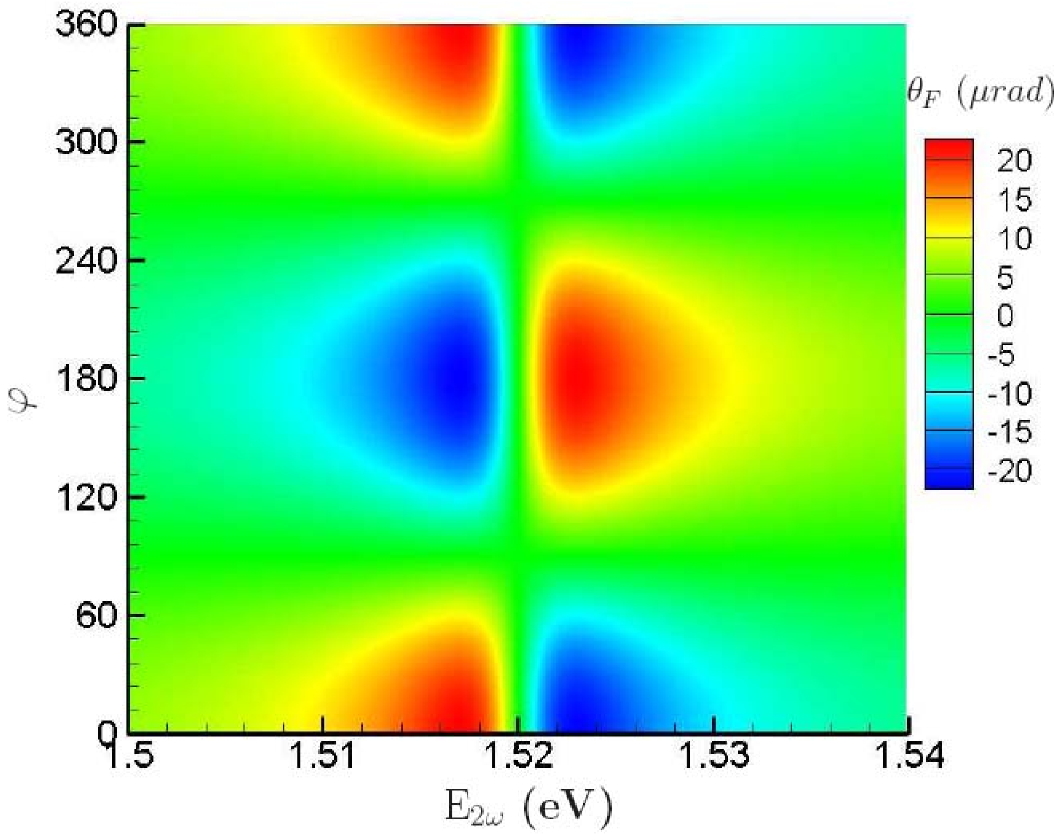
Contour map of the Faraday rotation angle θ_F (μrad) as a function of transition energy and the relative phase of the two fields $2\varphi_1 - \varphi_2$. The pure spin carriers is along the k_x direction.

Results and Discussion



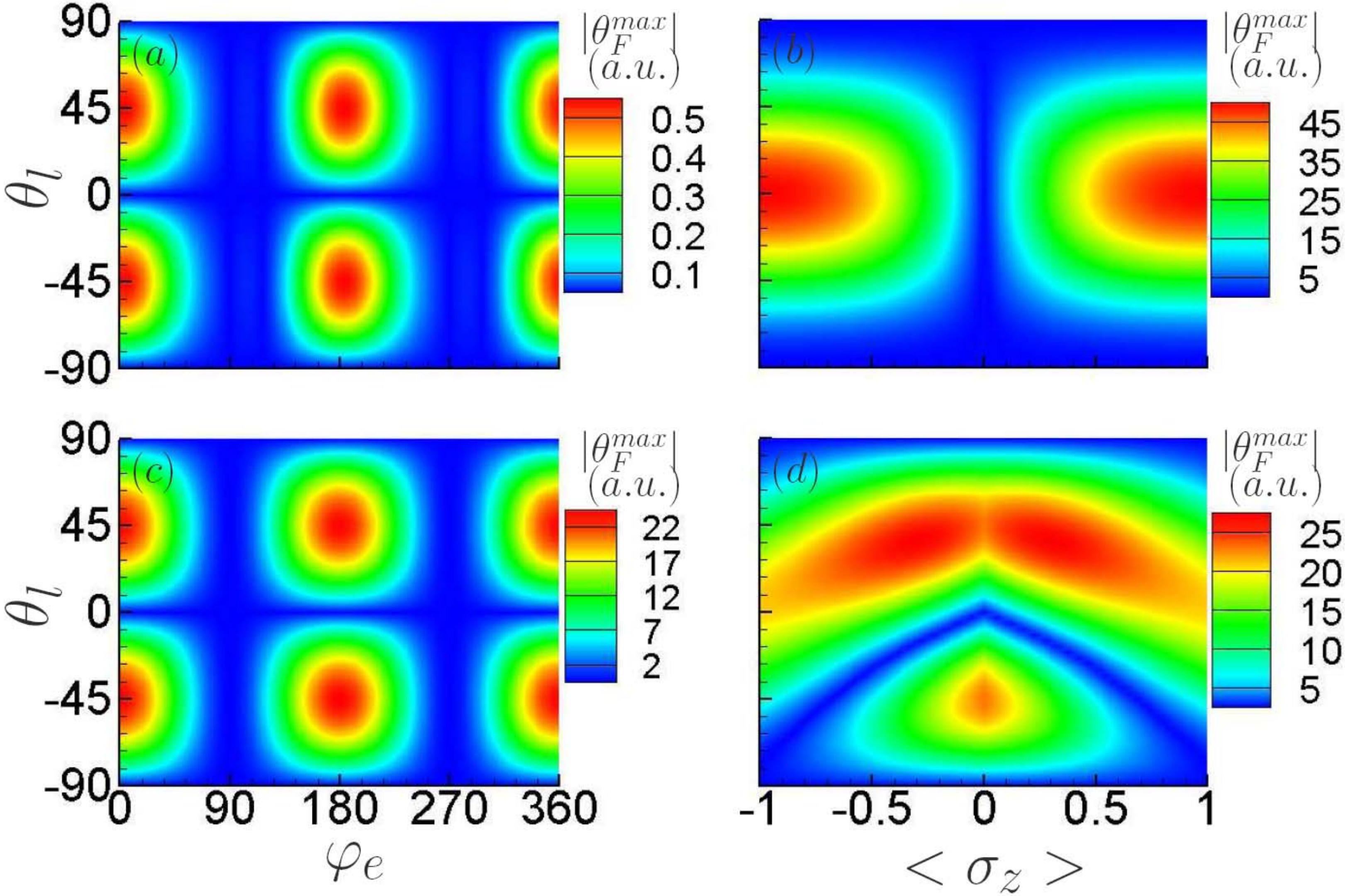
Contour map of the Faraday rotation angle θ_F (μrad) as a function of transition energy and the polar angle θ of the direction of pure spin carriers for $2\varphi_1-\varphi_2=0$.

Results and Discussion



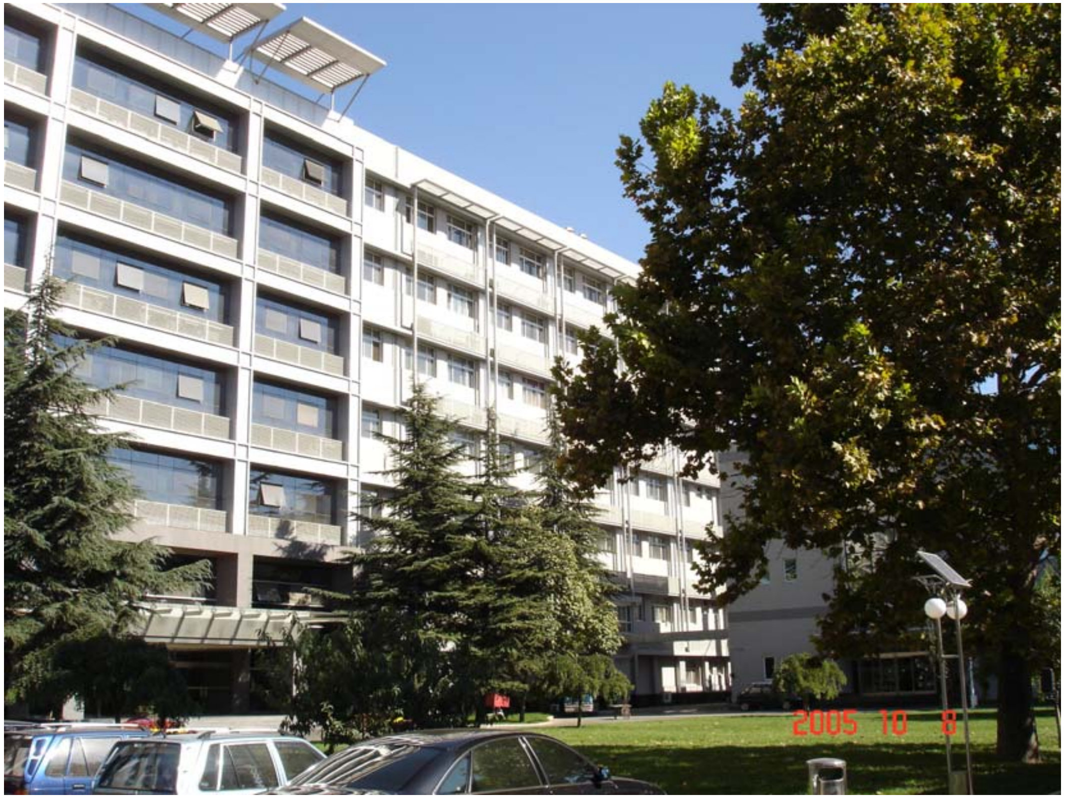
Contour map of the Faraday rotation angle θ_F (μrad) as a function of transition energy and the polar angle φ of the direction of pure spin carriers for $2\varphi_1 - \varphi_2 = 0$ and $\theta = 90^\circ$.

Results and Discussion



Conclusions:

Quantum interference Faraday rotation provides us a possible way to detect spin current directly, and help us to distinguish extrinsic and intrinsic Spin Hall effects.



Thank you for your attention!